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A Mechanical Model for Demonstrating Fermat's Principle

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An advantage of the *general principle* approach to physics as contrasted to the particularized formulation of physical relationships is exemplified by the solution of a research problem in ionospheric radio propagation. The generalized approach resulted in the construction of a mechanical analog computer which can be used to demonstrate Fermat's principle. This paper outlines briefly the theory of *least action*, shows how it can be applied to mechanical systems, electrical circuits, and ray optics to obtain the more familiar laws describing their behavior, and describes the essential similarity between an optical system and its mechanical analog. The mechanical demonstrator can be assembled from common laboratory equipment for classroom demonstration, or it can be carefully constructed for use in computing tables of radio propagation times for ray paths bounded by the earth and ionospheric layers.

THE purpose of this paper is twofold: first, to illustrate an advantage of the *generalized* or *fundamental concept* approach to physics as distinct from its dismemberment into algebraic descriptions of isolated phenomena; and, second, to present a demonstration device which illustrates an essential similarity of behavior between a mechanical and an optical system.

The demonstrator to be discussed was developed originally for use as an analog computer. The problem was to solve for radio propagation times over long ionosphere paths which, because of difficulties caused by the lack of symmetry among the path elements, could not easily be handled by ordinary geometrical methods. The model was fabricated as a part of the ionosphere studies using V-2 rockets being carried out by the Boston University Upper Atmosphere Research Laboratory under an Air Materiel Command research contract.[1]

The design of the model was prompted by a consideration of the fundamental properties of the transmission of radio signals between the terminals of a path having any configuration and the corresponding properties of mechanical systems. Figure 1 shows typical path configurations to be analyzed over a range of rocket altitude h and ionosphere virtual heights h', h_1', and h_2' with the total angle between the terminals remaining fixed.

Let us first consider the properties of a generalized system in which energy is being transported or interchanged between its kinetic and potential forms. The steps in this development can be outlined as follows:

1. Maupertius-Euler principle of least action:

$$\delta \int_{t_1}^{t_2} 2T dt = 0, \qquad (1)$$

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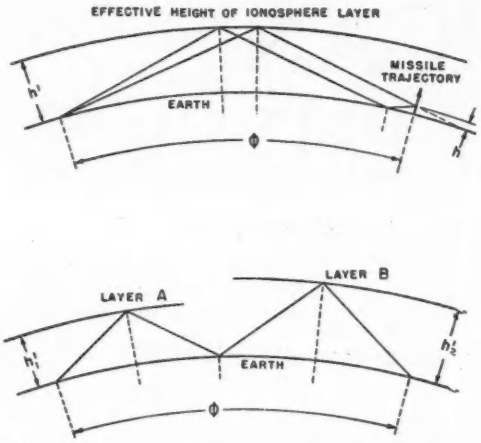

FIG. 1. Nonsymmetrical ionosphere ray path elements. Variation of the angles as a function of the ionosphere virtual height h' and rocket altitude h makes the application of trigonometric laws difficult. The angle between terminals ϕ remains constant.

where T is the kinetic energy, δ is the variation, and t is the time.

2. Hamilton's principle:

$$\delta\int_{t_1}^{t_2}(T-V)dt=0, \tag{2}$$

where V is the potential energy and $(T-V)$ is the kinetic potential (Lagrange function).

3. Generalized Hamilton's principle:

$$\delta\int_{t_1}^{t_2}(T-V-D+W)dt=0, \tag{3}$$

where D is the dissipated energy and W is the received energy.

4. Conditions for stability: The change in kinetic energy is less than the change in potential energy, which, with dissipation, degenerates to a state having a potential energy minimum (stationary value).

In a conservative mechanical system free to pass from one configuration to another, the motion is such that the *action*, defined by

$$A=\int_{s_1}^{s_2}mvds=\int_{t_1}^{t_2}mv^2dt=\int_{t_1}^{t_2}2Tdt, \tag{4}$$

is a minimum (or stationary) value, where A is the action, s the space integration variable, t the time integration variable, v the velocity, T the kinetic energy, and m the mass involved. This relationship is based on a free interchange between kinetic and potential energy and is known as the *Maupertius-Euler principle of least action.* It may be expressed symbolically as

$$\delta\int_{s_1}^{s_2}mvds=\delta\int_{t_1}^{t_2}Tdt=0, \tag{5}$$

where δ indicates the variation of the value of the *total* integral and mv and T represent the momentum and kinetic energy, respectively. This is a sort of *law of conservation of energy* which expresses Nature's tendency to expend the least possible amount of energy in going from one configuration to another or in transporting energy from one place to another. The action followed is the *natural track* or *geodesic* through four-dimensional space-time.

When both kinetic *and* potential energy in a conservative system are taken into account, we find the interchange of energy taking place in a manner such as to make the time integral of the *difference* between the kinetic and potential energy over the process a minimum or stationary value. This frequently encountered difference is called the *kinetic potential*, or the *Lagrange function:* $(T-V)=L$. Hamilton's principle, Eq. (2), is the mathematical statement of this relationship. It can be derived from Lagrange's equation, which expresses the same principle in the form of a differential equation.

If we include dissipation of energy from the system and receipt of energy into the system, the sum of the kinetic energy T and potential energy V at any instant is equal to the original energy U_0 decreased by the dissipated energy D and increased by the incoming energy W:

$$T+V=U_0-D+W. \tag{6}$$

The resulting generalized Hamiltonian principle can be expressed as

$$\delta\int_{t_1}^{t_2}(T-V-D+W)dt=0. \tag{3}$$

If it so happens that in a specific dissipative system the kinetic energy becomes zero for a given configuration for which the potential en-

ergy has a minimum, then the system will come to rest in that configuration. This approach permits complete generalization in the analysis of a mechanical system, and suggests certain analogies which might be overlooked if a more restricted interpretation were placed on the operation of the system.

A simple application of the principle of least action to electrical circuits yields *Kirchoff's law*. In a circuit consisting of two resistors R_1 and R_2, in parallel, the total current I flowing is the sum of the individual currents i_1 and i_2. This may be written as

$$I=i_1+i_2, \tag{7}$$

from which we obtain

$$i_2=I-i_1. \tag{7a}$$

The heat developed in each branch is i^2Rt, and the total heat E developed in both branches in time t is

$$E=(R_1i_1^2+R_2i_2^2)t. \tag{8}$$

Substituting for i_2 its value from Eq. (7a) we obtain

$$E=[R_1i_1^2+R_2(I-i_1)^2]t. \tag{8a}$$

Taking the variation with respect to the system variable i_1, and applying the principle of least action gives

$$\partial E/\partial i_1=2R_1i_1-2R_2(I-i_1)=0, \tag{9}$$

from which we get the familiar form of Kirchoff's law:

$$R_1i_1=R_2i_2. \tag{10}$$

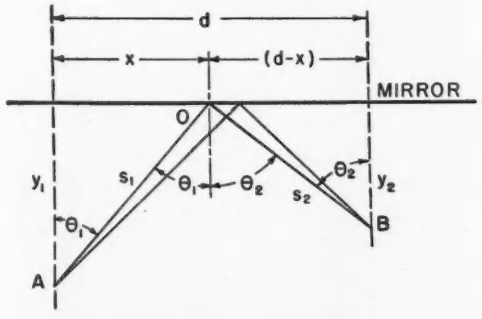

FIG. 2. Point of reflection determined from Fermat's principle. The length of path between the fixed points A and B is to be minimized as a function of the position of the reflection point O.

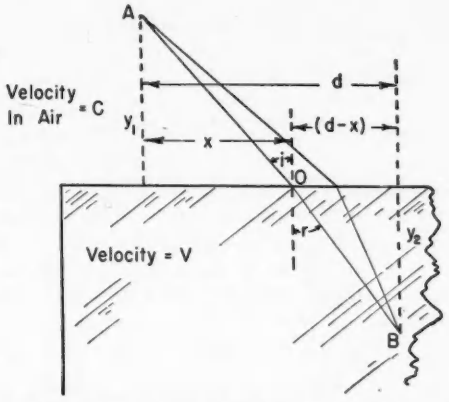

FIG. 3. Snell's law from Fermat's principle. The transit time over the path from A to B is to be minimized as a function of the velocities in the two media and the point of refraction O.

If we apply the principle of least action to the propagation of light we obtain *Fermat's principle*. If E is the radiant energy being transported at a constant rate and t the variable integration limit, the principle of least action may now be expressed as

$$\delta\int_{t_1}^{t_2}Edt=E\delta\int_{t_1}^{t_2}dt=0. \tag{11}$$

If u is the velocity of light in the medium and ds the element of path length, then $dt=ds/u$, and we obtain Fermat's principle in the form

$$\delta\int_{s_1}^{s_2}ds/u=0. \tag{12}$$

Further specialization in the field of optics results in the particularized *law of specular reflection*. Referring to Fig. 2, light travels from the source located at A to the point of reflection O on the smooth mirror and thence to the fixed point at B. The location of the point O is assumed to be unknown, but is defined by the distance x from A to the normal to the mirror erected at O.

The path-length elements s_1 and s_2 may be expressed in terms of the sides of right triangles, and the total path length then is given by the expression

$$S=s_1+s_2=(y_1^2+x^2)^{\frac{1}{2}}+[y_2^2+(d-x)^2]^{\frac{1}{2}}. \tag{13}$$

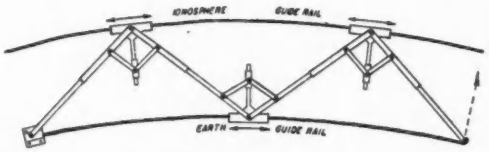

FIG. 4. Equal angle simulator based on law of reflection. Telescopic arms representing ray path are constrained to form equal angles at reflection points by sliding parallelograms.

Since $t=S/c$, we wish to have t meet the condition that

$$dt/dx=(1/c)(dS/dx)=0. \tag{14}$$

Taking the derivative, equating it to zero, and transposing, we obtain

$$x/(y_1^2+x^2)^{\frac{1}{2}}=(d-x)/[y_2^2+(d-x)^2]^{\frac{1}{2}}; \tag{15}$$

but by definition:

$$x/s_1=\sin\theta_1 \quad \text{and} \quad (d-x)/s_2=\sin\theta_2.$$

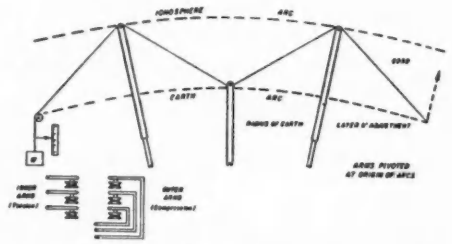

FIG. 5. Least time simulator based on Fermat's principle. Minimum potential energy of weight W corresponds to minimum length of ray path for all configurations of reflection points.

Hence we obtain the familiar form of the *law of reflection*

$$\theta_1=\theta_2. \tag{16}$$

The complication of the path caused by the introduction of several values of the index of refraction is easily handled to yield *Snell's law of refraction*. Referring to Fig. 3, we set up the elements of path length in the same manner as for the case of reflection, with the modification that we must keep track of the different values for the velocity of light, c in air and v in the medium, in the two portions of the total path. The total transit time from A to B by way of an unknown refraction point O will be

$$t=[y_1^2+x^2]^{\frac{1}{2}}/c+[y_2^2+(d-x)^2]^{\frac{1}{2}}/v. \tag{17}$$

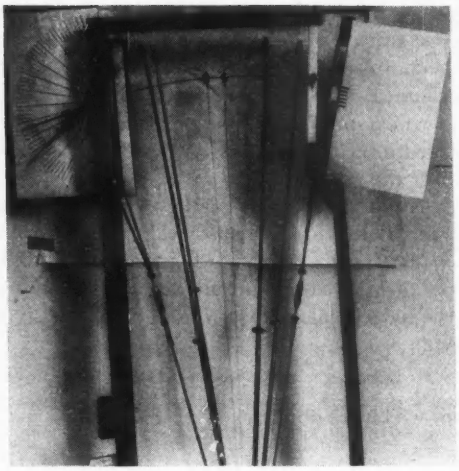

FIG. 6. Mechanical demonstrator for Fermat's principle. Counterbalanced telescopic radius arms constrain ray paths to reflection at earth and ionosphere. Change in position of pointers for each of two paths measures change in path length as a function of rocket altitude. "Two-hop" rays with final ground reflection on one path.

Performing the same operations of differentiation with respect to the position of the point of refraction O as above and introducing the trigometric functions of the angles we obtain

$$(\sin i/c)-(\sin r/v)=0.$$

Transposing gives the familiar form of *Snell's law of refraction*

$$\sin i/\sin r=c/v=n. \tag{18}$$

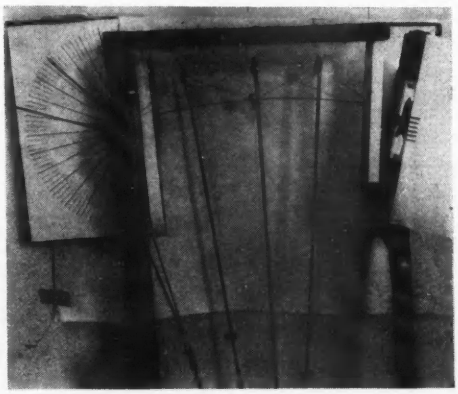

FIG. 7. "One-hop" (front) and "two-hop" (rear) rays displayed simultaneously. Rack and pinion driven rocket at ground position.

We are now in a position to consider the construction of a mechanical model whose principle of operation shall be analogous to the principle which describes the path taken by a radio ray in traversing the distance between our terminals by way of the undetermined points of reflection from the ionosphere and the earth. If we apply the limited concept of specular reflection, in which the angle of incidence equals the angle of reflection, we may arrive at a design such as that shown in Fig. 4.

The parallelograms and guides assure equal angles at the simulated points of reflection, while the combined length of the telescopic arms is a measure of the path length. The spacing between rails must be adjustable to accommodate various ionosphere heights, and the simultaneous demonstration of two or more paths becomes very difficult. The "simple" concept is seen to lead to a relatively complicated mechanical model having seriously limited flexibility.

If, however, we consider the more generalized concepts of the problem of propagation over a complicated path between fixed terminals, we soon arrive at a mechanical model designed along the lines of Fig. 5. The cord, under the tension of the weight W is constrained to touch the planes of reflection by the pulleys which, in this particular problem, lie on the surface of concentric spheres defined by the radius arms. The arms are freely pivoted and counter balanced. Displacing the origin of the arms simulates the condition of a nonconcentric ionosphere or "tipped" ionosphere layers.

According to our analysis, this model will assume that rest configuration in which the *potential* energy is a minimum, since in that position there will be no motion of the weight and the kinetic energy will be zero. The potential energy will be a minimum when the weight is at its lowest position, corresponding to the minimum length of string between the terminals. The ray path thus defined then bears a one-to-one geometrical correspondence to the path followed by a radio ray in traversing the equivalent configuration of terminals and reflection points. We have set up a mechanical demonstrator for Fermat's principle.

For classroom demonstrations, the model can readily be constructed from pulleys, meter sticks, string, and a weight. The surfaces of constraint may take forms other than those here employed. The model finally constructed permitted simultaneous display of two path configurations and modification to show a variety of combinations of reflection points and angles of reflecting planes. The model will reproduce *three-dimensional multi-reflections*, since the string need not lie in a plane to satisfy the minimum potential energy relationship.

Figure 6 shows the model set up to display the effects of two paths each having two points of reflection from the ionosphere, but one having an additional reflection from the ground before arriving at the rocket, which is shown at maximum scale altitude. The telescopic arms are counterbalanced and adjustable for ionosphere virtual height. The condition displayed has been made more complex than that shown in Fig. 1 by the addition of an additional ionosphere reflection. The condition of simultaneous paths encountering one and two ionosphere reflections, respectively, is shown in Fig. 7, in which the rocket is at the ground level on its rack and pinion driven guides.

When used as an analog computer for determining *changes* in propagation path length during rocket flight, as compared with a simple geometrical configuration which can be calculated for the zero-altitude case, the model has a useful accuracy of better than one part in one thousand of the total path length for all configurations. This accuracy equals or exceeds the accuracy with which experimental data can significantly be scaled.

The experimenter in physics, whether he be a professional mathematician or not, in one important sense must also be regarded as a theorist; if he were to describe the reasons which prompt his experiments, or the results which he has obtained from them, he would be making use of language which is without precise meaning except within the scheme of theoretical physics in general use at the time.
—NORMAN FEATHER, Lord Rutherford, 1940.

Some Analogous Problems Involving Variable Mass, Moment of Inertia, Inductance, and Capacitance*

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Four problems, a, b, c, and d, are proposed which deal with the application of (a) constant force to a mass m, (b) constant torque to a body with rotational inertia I, (c) constant emf to a condenser with capacitance C, and (d) constant emf to a coil with inductance L.

In (a) and (b) the acceleration is prevented by having the mass m in one case and moment I in the other increase at the proper rate. In (c) and (d) the current is kept constant by having the capacitance in one case and the inductance in the other increase at the proper rate. The power required is shown to be constant in each case and the distribution of energy supplied to each element turns out to be equally divided between kinetic energy, in (a) and (b), or field energy, in (c) and (d), and some system of restraints such as friction, springs, etc., which inevitably opposes the required change of the elements, m, I, C, and L.

SOME years ago Professor Sleator[1] proposed an interesting problem involving a force acting with constant horizontal velocity so as to set in motion an ever increasing section of rope as it is drawn off a coil resting on a smooth floor. Assuming that the only power required of the source goes to increase the kinetic energy of the system, we have $P_k=fv=d(\frac{1}{2}mv^2)/dt=\frac{1}{2}v^2dm/dt$, where f is the constant force, v is the constant velocity, and m is the mass of rope in motion. If, on the other hand, no assumption is made regarding the ultimate form of the energy developed and if losses due to friction and energy of extension are neglected, $P=fv=d[(mv)v]/dt=v^2dm/dt$, which is just twice that obtained by the first method. Account for the discrepancy.

The continuing interest in this intriguing problem is well attested by the articles and discussion notes[2-6] which have appeared in this journal in the past fourteen years. In these papers the authors show that only half of the energy supplied by the source goes to increase the kinetic energy of the system, the other half being required to supply the energy lost to heat resulting from the inelastic collisions of the moving portion of the rope with each element of rope as it is set in motion. It is the purpose of this article to discuss the appearance of a similar effect in three analogous situations. In this we shall compare the powers rather than the forces, as was done in the articles mentioned.

1. Variable Moment of Inertia

In its most general form the problem may be stated thus: A constant torque T is applied to a body rotating about a fixed axis with a constant angular velocity ω. The angular acceleration which would normally result is prevented by increasing the moment of inertia of the system at a rate dI/dt. In terms of ω and dI/dt express the power required and account for the distribution of energy that results. Neglect friction.

The moment of inertia may be increased by having the body pick up matter as it revolves, by moving the particles of the body outward from the axis, or by both methods. The first raises the problem of impacts again. Since this has been discussed in the articles mentioned, we shall consider here only the problem of a changing moment of inertia due to a changing distribution of the particles. This amounts, of course, to an expansion of all or part of the body. A set-up which is easily visualized might consist of a wheel, as in Fig. 1, provided with radial spokes upon which particles slide outward, thereby increasing the rotational inertia. The power needed to increase the rotational kinetic energy of the system is given by

$$P_k=\frac{d}{dt}(\tfrac{1}{2}I\omega^2)=\tfrac{1}{2}\omega^2\frac{dI}{dt}. \qquad (1)$$

But since the fundamental definition of torque is

* Contribution no. 8, Department of Physics.
[1] W. W. Sleator, Am. Phys. Teacher **3**, 138 (1935).
[2] R. T. Birge, Am. Phys. Teacher **4**, 43 (1936).
[3] W. W. Sleator, Am. J. Phys. **15**, 474 (1947).
[4] H. M. Dadourian, Am. J. Phys. **16**, 344 (1948).
[5] G. F. Hull, Am. J. Phys. **16**, 447 (1948).
[6] R. J. Stephenson, Am. J. Phys. **17**, 224 (1949).

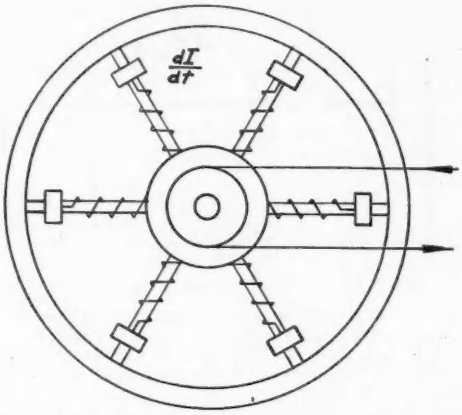

FIG. 1. A wheel is driven on a fixed axis by a constant torque T. The wheel is provided with radial spokes on which particles slide outward thereby increasing the moment of inertia of the system at a rate dI/dt which maintains the angular velocity ω constant. In terms of ω and dI/dt, what power is required? Account for the distribution of energy supplied to the wheel.

$d(I\omega)/dt$, the total power required is

$$P = T\omega = \omega\frac{d}{dt}(I\omega) = \omega^2\frac{dI}{dt}, \qquad (2)$$

which is twice P_k. Evidently only half of the power is used to increase the rotational kinetic energy of the system. What then becomes of the other half?

At any instant after constant angular velocity has been established, let us resolve the force on a particle of mass m_1 into its tangential and radial components. The tangential force $f_1 = m_1\omega dr_1/dt$, and the power required on account of this component is $P_k' = f_1 r_1\omega = m_1\omega(dr_1/dt)r_1\omega$. Hence, $P_k' = m_1 r_1\omega^2 dr_1/dt$. Also, since $I_1 = m_1 r_1^2$,

$$dI_1/dt = 2m_1 r_1 dr_1/dt. \qquad (3)$$

By combining these equations and eliminating dr_1/dt, one obtains $P_k' = \frac{1}{2}\omega^2 dI_1/dt$. Then, since the moment of inertia of the whole body is the sum of the moments of the parts, $P_k = \frac{1}{2}\omega^2 dI/dt$ which, as before, is the power required to generate rotational kinetic energy.

The radial component of the force on a particle m_1 is made up of two parts, one of which is the centripetal force given by $f = -m_1\omega^2 r_1$. By the time constant angular velocity ω has been reached, an additional central force is required to reduce the outward velocity of the particle in accordance with the conditions of the problem. For if P_1 is the total power required by the particle, $P_1 = T_1\omega = \omega^2 dI_1/dt$. Therefore dI_1/dt is constant because of the conditions of constant torque and constant angular velocity. Then by Eq. (3) dr_1/dt is inversely proportional to r_1 and the acceleration of the particle is negative. This force, obviously, is not supplied by the external source. Whether furnished by friction, by cohesion, or by the spring, this decelerating force merely converts the kinetic energy of radial motion of the particle into some other form. The only power requirement of the source, then, for radial motion is to stretch the spring against the centripetal force f_1.

Assuming then that the spoke alone supplies the radial working force, the power required of it is $P_r' = m_1\omega^2 r_1 dr_1/dt$. By use of Eq. (3) and summing for all the particles, we have the power producing radial motion $P_r = \sum f_1 dr_1/dt = \frac{1}{2}\omega^2 dI/dt$ which accounts for the rest of the energy as required by the momentum equation. Here, as in the rope problem, only half the power supplied by the source goes to increase the kinetic energy of the system which in this case is rotational. The rest goes to stretch the springs, produce heat, or do work against any other restraints that may exist.

2. Variable Capacitance

Figure 2 shows a variable capacitance connected in series with a source with constant emf E. The resistance is negligible. Let the capacitance be increased at a rate dC/dt such that the current I remains constant. (a) What power is required of the source? (b) What becomes of the energy supplied?

If one assumes that all of the power increases the electric field between the plates of a condenser, we have $P_k = d(\frac{1}{2}E^2C)/dt = \frac{1}{2}E^2 dC/dt$. But whatever the ultimate distribution of the energy supplied by the source, the power is given by

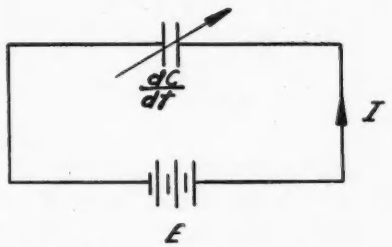

FIG. 2. A variable capacitance is connected to a source of constant emf E. If the rate of change of capacitance dC/dt is just sufficient to keep the current I constant, what power is required of the source and what becomes of the energy supplied?

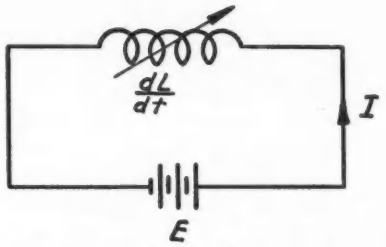

FIG. 3. A variable inductance is connected to a source of constant emf E. The current is kept constant by increasing the self-inductance at the proper rate dL/dt. What power is required of the source? Account for the distribution of energy supplied.

$P=EI=EdQ/dt=Ed(EC)/dt=E^2dC/dt$, which is twice P_k.

Textbooks on theoretical physics[7] show that whenever electrostatic forces do work in changing the configuration of charged bodies at constant potential, only half of the energy appears in the field. It is usually assumed that the rest of the energy appears as heat, change in kinetic or potential energy, etc., depending upon what restraints of friction, inertia, etc., exist in the system. It is instructive to evaluate the forces involved in a particular case and show that this is true.

For our example, let the capacitance be furnished by a large parallel-plate condenser with negligible edge effects. Its capacitance is increased by letting the plates approach at a rate $-dr/dt$ thereby doing work against friction or any other restraint at the rate

$$P_2=f(-dr/dt)=2\pi Q^2(A/k)(-dr/dt), \quad (4)$$

wherein $Q=EC/A$, the charge per unit area of condenser, A is the area and k the dielectric constant. Then since $C=kA/4\pi r$, $Ak=4\pi rC$ and $-dr/dt=(r/C)(dC/dt)$. By substituting in Eq. (4), one obtains $P_2=2\pi Q^2(A/k)(-dr/dt)=(\frac{1}{2}E^2)(dC/dt)$, thus accounting for the other half of the power involved.

3. Variable Inductance

Let a variable inductance be connected to a source of constant emf E, as in Fig. 3. The current is kept constant by increasing the self-inductance L at the rate dL/dt. What power is required of the source? Account for the distribution of energy supplied.

The power to increase the magnetic field is given by $P_1=d(\frac{1}{2}LI^2)/dt=(\frac{1}{2}I^2)(dL/dt)$. But the total power is $P=EI$, and since $E=d(LI)/dt=IdL/dt$, it follows that $P=I^2dL/dt$. Again, the power supplied to the field is only half of the total power involved.

[7] Page, *Introduction to Theoretical Physics* (D. Van Nostrand Company, Inc., New York, 1928), pp. 341–42.

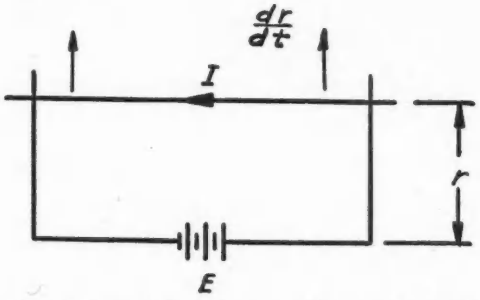

FIG. 4. The variable inductance of Fig. 3 might consist of a pair of fine parallel conductors, one of which is free to slide away from the other at a rate dr/dt, thereby increasing the inductance of the circuit and maintaining the current constant.

As in the other cases, the general theory shows that half of the total power is required to do work against whatever restraints exist which may oppose the change in inductance. For example, the circuit might consist of a pair of long parallel conductors connected in series to a battery of emf E as shown in Fig. 4. Since the currents pass in opposite directions, there are forces of repulsion which tend to move the wires apart, thus increasing the self-inductance of the circuit. Let the wires move apart at a rate dr/dt such that the change in inductance is just sufficient to maintain the current I constant. The force pushing the wires apart is given by $f=2\mu I^2l/r$ in which μ is the permeability, l the length of the wires, and r the distance from center to center. The mechanical power required is

$$P_2=fdr/dt=2\mu l(I^2/r)(dr/dt). \quad (5)$$

The self-inductance of the parallel wires is given in textbooks on electricity and magnetism by the equation $L=4\mu l\log_e r$, if the diameter of the wires is negligible in comparison with their distance apart. On taking the derivative of L, $dL/dt=4l(\mu/r)(dr/dt)$. Eliminating dr/dt from Eq. (5), $P_2=(\frac{1}{2}I^2)(dL/dt)$, thus accounting for the balance of the power required of the source.

Doubtless other problems may be found which are analogous to the ones proposed here. In acoustics, one would expect an element involving a variable acoustical capacitance or a variable inertance to behave much like a variable electrical capacitance or inductance. Possibly, too, there is a similar analogy in radiation. A discussion of these analogies may be undertaken in a subsequent paper.

Teaching by Publication*

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(Received June 26, 1950)

Remarks upon the present shortcomings of publications dealing with physics at both scholarly and popular levels of exposition, with suggestions for improvement.

THERE is a tendency nowadays to distinguish too sharply between research and teaching; and there are even some physicists who feel that if they are hired to do research work, they are thereby emancipated from teaching. It is part of the purpose of this article to redirect such misguided persons by pointing out that publication of research results is also a kind of teaching.

The New York *Times* of June 3, 1950, presented on its editorial page, in a gently satirical form, a keen comment concerning the impact of new modes of publication upon what might be called face-to-face teaching. We may fully agree with the unidentified professor, quoted by the columnist, that there is often need for immediate give and take between teacher and taught, and that those who must miss direct exchanges of question and answer with those who know more miss a great deal. There are wide differences of opinion regarding the benefits of this kind of teaching to the teacher; and, so far as I am concerned, what I get from twenty sophomores in 3000 minutes per year is not indispensable. It is largely due to the need for teaching people who are not sophomores and cannot be coerced by college administrators to assemble at stated times and places that teaching of other kinds also seems to me to be important.

Publication

The act of publication is impossible unless there exists a public, a group of people not sharply delimited from all others by a rigorous selection rule of any kind. A private letter to a single addressee is not a publication, and neither is a telegraphic message to all naval stations. A publication is a document which is open to all who are interested. This is not to say that a publication may not be addressed to an individual or designed otherwise to influence a single agent, like a church or state. Letters to an editor in scientific controversies are familiar examples of publications which are aimed at a particular opponent. Reports to boards of regents, to sponsors of experimental programs, to stockholders, etc., may or may not be published, but very often are.

Some people confuse multiplication of copies with publication, but there is no necessary correlation between these. During World War II, I helped to prepare and issue a monthly top-secret list of priorities for bombing in the Japanese area. Though the number of copies sent out, by official messenger, was around three hundred, this periodical was not a publication. At the other extreme we have the wooden post on the eastern side of the New Haven green. A single copy of a legal notice tacked to this post has been published and is open to all, in spite of the obvious fact that this is an inadequate way of advising all citizens of what has happened or is about to happen. When Martin Luther posted his theses on the church door in Wittenberg, in the year 1517, he published them in precisely this way.

The facts just considered relieve me from the need for discussing many subjects which are usually belabored when the problem of adequate publication is presented to scientific readers. Nothing about dues, subscription rates, printing costs, or support of periodicals by advertising or by subvention need be included. As publishers, we will be satisfied as soon as one perfect copy of a communication is available to the most ingenious and inquisitive members of its public. This is all the more allowable if we are aiming to teach just these already well-taught people. It is suggested that any principles which we may find applicable to immediate publication should

* Based upon an after-dinner speech at the Middletown meeting of the AAPT, June 20, 1950.

also be applied to delayed publication, that is, to the preparation of documents which are temporarily to be classified as restricted, confidential, secret, top secret, or the like, but which may later become public.

Publication in Physics

So far we have not confined ourselves to physics, or even to science, but must now do so, because the chief difficulty in teaching physics by publication arises from the nature of this science. Anyone may devise a definition of physics to suit his need, and for the moment we may try the following:

> Physics is a group of general statements about matter, energy, space and time, consistent with observations.

It may turn out that we can do with fewer than these four "elements," in the philosophical sense; but if and when this occurs, the acceptable statements in the earlier physics of that day will be translated into acceptable statements in the physics which replaces it.

You will notice that our definition of physics confines it to *general* statements consistent with *observations* and the *one* statement, in order to be *general*, must be consistent with more than *one* observation to win admission to our subject. This means that a single datum, however valuable, is not physics but only part of the fodder upon which physics feeds and grows. Even with this restriction we have an embarrassment of riches in all the valid general statements that can be devised. There must be some least number of such statements that imply all the rest, and a good New England fundamentalist, like Professor Bridgman, can get a lot of quiet fun out of lopping away redundant links between statements actually anchored to observations and those more remote from such bases. However amusing this pruning process may be, it is of little use to us, poised as we are between the devil of facts and the deep sea of language, into which, of course, we must plunge when we publish.

Publication for Large Groups

That part of teaching by publication which does not profess to teach experts is essentially a selection among the valid statements of physics which either "covers the waterfront" in a general textbook or tries to include all the pebbles on a limited part of Sir Isaac Newton's beach in an encyclopedic review article. It is a great pity that both of these jobs are usually botched by the very natures and nurtures of the people who undertake them.

The Textbook

The textbook is usually written by a successful teacher, or by a group of such teachers. Since the successful teacher excels in the give-and-take style of teaching, he is likely to be rather less effective in uninterrupted exposition. Moreover, he is likely to be tolerant of well-meant but not quite exact statements, which he hears so often in the classroom and must not, there, criticize too curtly. Every elementary textbook contains statements which are invalid in detail, even if they would rate full marks in a ten-minute quiz. Many of these apparently result from a mistaken *esprit de corps* which, for example, prevents the professor responsible for Chapter VI from red-penciling the manuscript of the professor responsible for Chapter XVI. Another fruitful source of boners in textbooks is the habit of building each text in part from the scrap lumber of an older text, which, for obvious reasons, is out of date. The inspection of the salvaged timber for knots and nails is the least attractive part of the job and is often scamped.

A surprising thing about many books which carry the title "physics" and which have little enough space for that subject is the amount of extraneous prattle which they contain. The following quotations from a recent text which I will not further identify are examples of what I have in mind.

> "Great poets, artists, philosophers, preachers and scientists have all contributed to man's ideal of the good and the true and the beautiful. Dreams have given man his soul (or *vice versa*). His dreams are the meaning of his life."
>
> "And it is estimated that there are still in this country a million human eyes doing jobs that could be done better with the electric eye."
>
> "We can look for many blessings from atomic energy. But first we must learn to control the bomb, with its threat to the world."

I have no objection whatever to a physicist who tries to make the world better, but this is not the responsibility of physics, which merely strives to improve human understanding of the world as it is. Eighty-four years ago the *London, Edinburgh and Dublin Philosophical Magazine and Journal of Science* contained a good technical article[1] on the height of drop necessary in hanging to break the necks of men of various dimensions and weights. The paper contains no comments on the moral aspects of capital punishment. This is still a good model for scientists to follow. If we mix emotion with science, we may expect to attract into science some very ill-adapted recruits.

What I am really saying here is that good teachers for classrooms and lecture-halls should probably not be allowed to write textbooks at all. Books should be attempted only by people who are so good at expressing themselves on paper that they may be able to transmit ideas to students by "*absent* treatment."

Video

I have been assuming, tacitly, that students will always be able to read the printed words of absentee teachers. When video has its perfect work, this may cease to be true; but even then I will insist that the concoction of the physics television show should be the business of an artist who does not need the stimulation of a visible and audible roomful of people for best results.

I have not much constructive advice to offer for the betterment of textbooks on physics. What would help, but what I do not expect to happen, is a return to the heyday of the footnote. It would be refreshing to see a text in which the few essential statements appeared at the top of each page, covering perhaps ten lines at most, with the rest of the page frankly labeled as comments by the author, not presented as evidence, but added to guide classroom discussion.

One further comment may be worth making. All textbooks I know are arranged to follow the history of the subject too closely. In a day when the fundamental nature of electricity is apparent, physics textbooks still start with the mechanics of antiquity in order to bow, in an early chapter, to an effigy of Newton and confuse the student thoroughly with systems of units which were invented to impose a logical structure upon the originally arbitrary and separate units of Scotch engineers. I still confidently await the appearance of a system which will "rationalize" the horsepower. If we started the physics of particles in the place where it belongs, that is in electron guns and oscilloscopes, and worked up to macroscopic mechanics as a special case for the slow motions of heavy projectiles at velocities not more than a few miles per second, a generation of physicists and engineers might develop with intuitions of some practical use. Heat might even be approached via thermodynamics instead of requiring an eighteenth century detour via Joseph Black and the phlogiston theory.

If the first opportunity came early instead of late in a year of elementary physics, we might get young people to deal habitually with dual presentations of phenomena by training them to comprehend both particles and waves in small scale physics. Why should we deny them this power just because the notion is twenty-five years old rather than five hundred?

Reviews

The review article is, in this country, usually written by an authority in the subject reviewed; but the hitch is that the writer often is really expert only in part of the field which he tries to cover. What happens, in consequence, is that the parts he knows best are biased in favor of his own work and of those who do him honor, and the parts he knows least are covered hastily and inadequately in order to complete his assignment. Nobody of consequence commits such crimes willfully; but, try as he may to be completely fair, a reviewer will be happier and more convincing in repeating what he knows well than in reporting what he has only just skimmed. These faults are usually so obvious that any conscientious abstractor will comment upon them; but this does not help very much, for the existence of one elaborate review, however faulty, makes another in the same area less likely for a while, and by the time a new review is justified,

[1] Samuel Haughton, "On hanging, considered from a mechanical and physiological point of view," *Phil. Mag.* (4) **32**, 23–34 (1866).

even if the same "authority" is not called upon again, the mere passage of time has so fogged things that work once slighted or misconstrued may never regain its fair share of credit. This, to be sure, rarely affects the future of physics; but it may be a little hard on the reputations of some physicists.

The professional reviewer has more vogue in other countries; and he, being expert in nothing but reviewing, can be very even-handed and do very useful work in presenting a good map of a recently annexed field of interest. Since he cannot usually go back very far, or translate jargon of the past into modern terms, he cannot write the epoch-making book in any subject. Such a work must be written by an expert just as he begins to run down, when he has most to gain by stimulating others, and least to gain by blowing his own horn. Unlike textbook writing, the writing of an epoch-making topical book is purely a labor of love, as it rarely sells well until it is out of print.

Even with the weaknesses just noted, the review and topical book literature of physics is much better on the average than its textbook literature and is even better than the shorter monographs which we will next consider. The longer works are, by their nature, time-consuming, and men live long enough with them to become dissatisfied with first drafts. Unlike papers which have emotional appeal, scientific papers are almost always improved by the author's revisions, and what is wrong with too many publications in physics is that not even their originators have had time to observe and eradicate faults.

Monographs

The harm done to physics by rambling and loquacious textbooks and by lopsided reviews is repeated and compounded in that kind of teaching by publication which tries to extend our subject by making *new* valid general statements about matter, energy, space, and time. Here, if anywhere, we should find clear simple statements, carefully separated from the new data, if any, and from the background of previously accepted statements and descriptions of standard procedures in experiment and analysis. This is just what we seldom find in our periodical literature. Bits of new truth are to be discovered in most of the papers which win publication, but it is far from easy to filter them out from the dust heap of second- and third-hand left-overs with which they are mixed. One sometimes suspects that the author has so recently picked up all his knowledge that all of it looks equally new and exciting to him.

Appeal for Action

So far, the tone of this paper has been depressingly negative; and, if this continued to the end, the objection might well be raised that it is no longer considered good pedagogy to teach by displaying the horrible example, as, we have been told, the Spartans of classical antiquity taught the value of a virtue by showing the result, upon human guinea pigs, of the corresponding vice. Fortunately, we can do better than this.

The appropriate action is to make our students of physics practice the art of writing physics rather than the art of writing physics mixed with a lot of other things. The training can begin with the first quiz of the first year and go on to the dissertation for the doctorate. This job cannot be done by professors of English, even if they can be interested in attempting it, because they cannot distinguish statements worthy of admission to our subject from statements completely foreign thereto. The result to be hoped for is a vast decrease in the wordage of physics, and in the time required to read both the generally uninteresting papers of our sophomores and the more stimulating papers of such physicists as may develop from them. This practice should also help to emphasize the difference between an exact science and pseudo-sciences which parrot the patter into which we have fallen in trying to make physics "popular." (This is no longer necessary.) To the extent that we have made physics sound like "Shakespeare and the musical glasses," we have lost its true elegance of conciseness and cogency, and must accept the well-deserved criticism that we seem to be trying to drum up recruits unfit for its active service.

A False Objective

One of the most debilitating ailments which now affect crusaders for the spread of physics,

even within its legal boundaries, is a fever of enthusiasm for the so-called scientific method. Perhaps some who read this have suffered at times from this affliction, which so warps the judgment of the victim that he is quite likely to argue that the justification for teaching physics lies not in its own values, but in the fact that it best exemplifies the scientific method. The implied assumption is that everyone should use the scientific method in all his work, an assumption contrary to fact. The wise nonscientist, who knows that there are good unscientific methods for many human tasks, sees the weakness of this argument for teaching physics, but is usually unable to express himself in sufficiently technical terms to correct our blunder without offending us. We may go just far enough with the unfortunate physicists who have been infected by this disease to admit that a knowledge of scientific method is worth having, but not so far as to admit that it should be taught instead of any science, including physics, which has a life of its own. If we do not combat this creeping paralysis of our subject, we may some day find ourselves in the same oubliette with those Latin and Greek scholars of an earlier generation who lost the contents of their subjects, and their students as well, by concentrating on grammar and syntax at the expense of the classics they should have taught students to appreciate.

Condensation

After we remove from current offerings to the literature of physics the impedimenta of other causes, we still have to condense and rearrange what is left to help the reader find what is new, or supposed to be new. If really good jobs of condensation and emphasis could be guaranteed, the job of abstracting, which is the rewriting, in even briefer form, of the new statements, would be easy, and keeping up with the literature of physics would be less like trying to swim up Niagara Falls. We used to rely entirely upon editors to prune papers submitted for publication, and they still do what they can; but the real place to use the shears is in the rooms where the original writing is done. It is a good rule to write a paper out, put it away for a week, and then revise it as if it had been written by a personal enemy, mercilessly excising the trite, the obvious, and the inconsequential, and straightening up every deviation from the straight and narrow way to truth. It is also an author's responsibility to write his own abstract. If the abstract is all that is really needed, may not the paper be dispensed with? If nothing at all is left, as may well happen, everybody will be happier.

In the process of condensation here advocated, it will often occur that a tubful of soup containing more than one oyster can eventually be served in two or three cups each containing a prize, and the chance of missing good things in too long a paper with too short a title will be much reduced. Incidentally, those whose scientific production is measured by counting titles in a president's report might even gain a little credit for making two blades of grass grow where one bale of hay might have dried up unnoticed.

Summary

In summary, then, I suggest that brief publication of original work and adequate condensation of other people's papers in reviews and textbooks are ways of teaching physics which deserve more care than they get and must get more care if physics is not to become a lock-step routine in which files of students must each do everything over again from the beginning because texts are stereotyped and libraries so distended that one cannot easily find suitable starting points for real advances. In original papers I recommend that we distinguish between references to prior publications, details of experiment, records of data, analytical steps, and conclusions believed to be new and justified. If parts of a research report seem of probable interest to different groups of readers, I advise its separation into true monographs.

Thus taking thought for colleagues, past, present, and future, in preparing papers for publication is only a part of our professional ethics; but it is one important part, as others[2] have recognized.

[2] Most recently: Ward Pigman and Emmet B. Carmichael, Science 111, 643–647 (1950).

The Old-Time Classroom Recitation: Can It Be Restored?*

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Large enrollments and relatively restricted teaching staffs have in recent years increased the challenge to teachers. But in addition to this, the diminished emphasis, that was generally adopted earlier in the century, on the recitation as a teaching method, has left limited opportunity, or none at all, for adequate oral self-expression in the classroom. Such training, it is felt, is indispensable to the student's broadest development, whatever other methods, however meritorious, may be used. The problem is serious and requires attention. It is discussed here in some detail.

The question is raised whether now, in too many institutions, the lecture method is overstressed, encouraging too much in students an habitual dependence on the lecturer, with a resulting lack of development of dependence upon themselves

CAN the old-time recitation method be restored? Should it be restored? What *is* the old time recitation method?

In 1911 a little book of about a hundred pages was published[1] with the title *The Recitation*. At that time this educational method was still in use. That the method, when properly applied, had its merits is evident from a perusal of the book's contents. Although it was addressed originally to teachers in the grades, its basic principles were applicable also to teaching on the college level. They are able to offer helpful hints to teachers of college physics today.

We pass from 1911 to 1928. In that year another book appeared with the title *The Passing of the Recitation*.[2] In more than three hundred pages, it cites the opinions of many leaders in education supporting this theme. Obviously, in 1928 the trend was definitely away from the recitation as an educational method or, at any rate, toward a modification of it.

What of the Present?

Professor Lloyd N. Morrisett, of the School of Education at the University of California, Los Angeles, recently stated that the recitation never did pass completely out of use in the grades and secondary schools, but only the *re*-citation phase of it, which was deliberately discarded.

He emphasized that the repetition, no matter how accurately, of mechanically learned material came to be recognized as falling seriously short of sound education. Today, he went on, what is termed "directed activity" has taken the place of the older teaching method known as "the recitation." He mentioned having observed not long ago an eighth-grade group engaged individually, under the teacher's direction and encouragement, in looking up different segments of the same general topic. They used encyclopedias and dictionaries, their results finally being pooled in what amounted to a sort of research seminar. Thus, the individual pupil is expected to develop nowadays by being put largely on his own resources, under competent supervision, to accomplish what he can individually. He then reports personally on what he has learned. His report is subjected to criticisms by classmates and supervisor. In this way his own view of the subject is widened.

The question how much improvement the introduction of directed activity has brought to teaching methods in elementary and secondary schools is not being raised here. But have we soft-pedalled perhaps too much any valuable feature of the old-time recitation?

Doubtless, before the passing of the recitation, able teachers saw to it that the individual pupil achieved something more than a mechanical repetition of facts mechanically learned. Skillful

* A paper on the program of the American Association of Physics Teachers, Southern California Regional Section, meeting at the California Institute of Technology, October 20, 1949. On this same subject also, W. W. McCormick's paper "The Neglected Recitation" should be read, especially for specific suggestions (See W. W. McCormick, Am. J. Phys. 18, 205 (1950). His article was unknown to me until it appeared in published form. However, there is no undesirable repetition here, but a somewhat different approach, and certainly agreement on the main theme.

[1] George Herbert Betts, *The Recitation* (Houghton Mifflin Company, Boston, 1911).

[2] V. T. Thayer, *The Passing of the Recitation* (D. C. Heath & Company, Boston, 1928).

questioning could always bring out statements showing how much the pupil understood of what he was saying. If the recitation was anything more than a verbatim repetition of what was memorized, the pupil must have got some training in self-expression, whether the ideas were fully understood or not.[3]

What suggested the present topic to the writer was the feeling that the individual college student could well use more opportunity for oral self-expression. In our lower division physics classes the present opportunity for such development is restricted pretty largely to written tests and laboratory reports. Opportunities for verbal discussion by students are limited practically to the laboratory. Here, the individual may seize an occasional chance away from the taking of measurements to talk with the instructor. To be sure, students are at liberty to come for questions and discussions during the lecturer's office periods, but the number taking advantage of this opportunity is relatively small. Such contacts hardly afford practice in the self-expression we mean; *viz.*, the standing before a group, speaking to them in lecture style, and answering questions.

Desirable Characteristics of the Recitation

The kind of recitation the writer has in mind is something more than a series of questions put to the student while on his feet before the class, that are answered briefly by yes, no, a short phrase, or an incomplete sentence. The recitation should be a lecture in miniature, where the student expresses himself in complete sentences of his own construction and is held responsible for making himself and his subject clear. I remember vividly an occasion during my student days when the instructor declined to aid me as I stood at a blackboard trying to explain Fresnel zones to my classmates.

Now this instructor, in refusing help at the moment, must have decided that the neophyte had enough general grasp of the basic principle to follow through by himself to some kind of logical conclusion, however haltingly and inadequately that might be done. To a student such treatment may seem at the time pretty rough. But a teacher can render no greater service to one of his charges, than to put him squarely on his own feet, so that he will *realize* he is on his own feet, with no chance to run away.

Is There a Present Over-Emphasis on the Lecture Method?

Nowadays, particularly in lower division physics, the restriction of opportunity for self-expression in the classroom goes along with the great emphasis that is placed on the lecture method. This method, in the writer's opinion, is admittedly valuable, even indispensable.[4] However, regarding the whole physics-teaching program, there would seem to be a present over-emphasis on this one method, an over-emphasis created by large enrollments, together both with limited facilities to cope with them, and with the large amount of material to be studied.

As to the latter, are we proceeding too fast with what may be called the "sifting-down" process? In this particular process, affecting especially today undergraduate teaching, topics that not so many years ago appeared only among research workers and in graduate seminars have been boiled down to simple forms that are finding their way into the general physics textbooks. This process is natural and inevitable. At the lower division level, is it proceeding too rapidly? Or, is it simply unavoidable?

The physics teacher cannot escape the fact that the reading public is familiar with newspaper articles and photographs having to do with betatrons and cyclotrons, with jet planes and rockets, etc., so that the alert student, when he reports for collegiate training, is ready with questions that need on the part of the instructor a sound acquaintance with the general facts of the newer physics. But the experienced physics teacher knows well that neither the newer physics nor the newer engineering can be really understood and appreciated without some mastery of classical physics. The teaching and the

[3] Does so-called "complete understanding" come immediately anyhow, or one might ask does it ever really arrive? A mark of 100 on a given examination is hardly 100 percent proof of 100 percent understanding of all the questions asked. A perfect mark on a bluebook—and such may occur occasionally—means only one thing: that the limited requirements of that particular test have been fully met.

[4] A useful feature of the lecture method, if adequate reading and criticism can be provided, is the lecture notebook. See J. S. Miller, Am. J. Phys. **17**, 582 (1949); also, L. E. Dodd, Am. J. Phys. **18**, 263–7 (1950).

thorough learning of the classical principles themselves can keep teacher and students fairly busy!

However, our beginning students come to us today with an outlook quite different from that which their fathers and grandfathers had. Somehow or other nature has endowed them with up-to-the-minute adaptability to conditions of life as they find it. To them automobiles, airplanes, the newer jet planes, and the numerous gadgets based on more recent discoveries are commonplace. They think in terms of them. When they meet in a physics textbook the statement that light is of dual character, waves and particles, the statement carries a certain naturalness and inevitableness to them, and they accept it, although as a total process its explanation is still incomplete even in the minds of eminent scholars. This receptivity of young minds is encouraging. If an ingredient in their adaptability is anything like a willingness to accept uncritically, then nature must have intended this as an aid to their learning. Thereby, also, nature laid upon us teachers responsibility.

Now and then a direct question may be asked that can prove embarassing to an instructor. It may come from the sudden penetrating insight of an able student, or from one who has learned experimental facts from his own experience, such as designing radio or television sets. The instructor must make clear to his physics students that their study is of basic *physics*, not engineering. The effort must be made to relate all questions to basic principles. No student's interest already formed, by whatever means, is to be dampened. Such *a priori* interest, even though primarily of an engineering character, is a teacher's opportunity. First, last and all the time a teacher is a guide. The student is to be guided to an understanding of the physics so that eventually he can answer his own questions. We do well to remember that nowadays, as in the past, it is usually the younger generation of physicists who create the new ideas that provoke progress.

It is the teacher, older in years and in knowledge of physics than his students, who undergoes special strain as he tries to keep up-to-date and bridge the space between the thinking of Archimedes, Newton, and others of former times, and that of Einstein and his contemporaries. The onrush of discovery and invention can hardly be stopped, although its control is advocated by some. As teachers we are not to overlook our own adaptability.

Valuable Features of Former Methods Should Be Used

Our main thesis is that any disused teaching method whereby the student could be helped effectively toward greater individual development, should be restored. If it should be, then we have a right to think it can be.

At the college level, when classes are limited to the lecture and the laboratory, a certain lack of balance develops. This is detrimental to the student, not only in that he gets less practice in oral self-expression, but also because his view of things in the classroom and to some degree in life becomes lopsided. There is missing the salutary discipline in which he himself, rather than the lecturer, is occasionally the star performer. The regular lecturer may on his part come to feel that he has, in the eyes of the class and in his own eyes, too much the character of a paid actor, critically watched by his audience for any possible slip in the performance. Thus among the students there can result a superficially critical, but in reality uncritical, attitude—from the standpoint of learning the principles of physics—that is too easily satisfied with what might be only a superficially appealing performance. They look for an uninterrupted flow of words and a smooth manipulation of apparatus. Only if student self-criticism is aroused adequately, is a sound balance in teaching attained. This desideratum the best aspects of the old-time recitation supplied.

One obvious solution where room scheduling and teacher load permit is to form special sections limited to an enrollment of about twenty students. It is, of course, assumed that participation by individual learners is guided competently. For best results in our present day physics teaching it is necessary to combine three distinct approaches to learning: (1) well-supervised oral expression; (2) laboratory experimentation; and (3) effective presentation of thoroughly prepared material in the lecture room.

Facing Our Problems

The present situation facing teachers of college physics is broadly similar to that facing the student as he confronts what to him may seem to be an impossibly difficult physics problem. By all means he is to be encouraged to read it through deliberately and carefully. This is the necessary start. Thereby he can be assured that even if the problem is not solved soon, at any rate a needful foundation has been laid that will make later progress possible, with appropriate help. That help, provided assistance from outside the student's own resources does not come too soon, may well be self-help through comprehension that dawns eventually from his own continued thinking about the problem. In any learner's mind there always has to be a beginning. He should be encouraged to make the beginning himself. Likewise, with the individual teacher facing *his* problem, help from outside his present circumstances may be delayed. He must then make the most of the situation.

It may be noted here that textbook writers often and rightly encourage independence by revealing answers to only a part of the problems. In order to learn the missing answers, it is natural for the student to turn to the instructor, who may need to use will power in denying this request. He must encourage the student toward increasingly firmer reliance on his own work.

It seems clear that in our large college classes we need more compulsion toward oral participation by the individual student, a broader and deeper stimulus to individual endeavor, to a greater ability, through practice, in accurate oral self-expression. Can a subject matter be found better adapted for the growth of such accuracy than physics?

Collisions of High Energy Nuclear Particles with Nuclei

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The experimental results for n–p and p–p scattering at high energies are described, and a review of the present status of theoretical interpretation is given. For the scattering of 90-Mev neutrons by protons the angular distribution indicates that only even states of angular momentum are strongly effective. Evidence of the operation of exchange forces is mentioned. In the scattering of 32-Mev protons by protons the angular distribution is appropriate to s-state interaction alone. The 340-Mev p–p results indicate the existence of noncentral (tensor) forces in states of odd orbital angular momentum.

I. Introduction

IN the first of this series of articles[1] dealing with research in high energy nuclear physics at the University of California Radiation Laboratory the characteristic features of the synchro-cyclotron, the synchrotron, and the linear accelerator were discussed. The present article will consider the nature of the effects observed when the high energy particles developed in the synchro-cyclotron collide with target nuclei.

A detailed discussion of the interactions between individual nucleons, and of the relation of present observations to nuclear force theories will be deferred until the next installment. Likewise, the production and properties of mesons will be treated later. Here the discussion will concern elastic and inelastic collisions of the accelerated particles with complex nuclei, and the variety of effects observed when a very energetic nuclear projectile enters a nucleus.

For a nucleon with kinetic energy in the hundreds of Mev the de Broglie wavelength is small compared to the dimensions of a heavy nucleus. Consequently, it is to be expected, and is observed, that the target nucleus does not always function in the collision as an entity itself; but rather that the momentum transfers may involve only one or a few of its constituent particles, and the energies given them may be large compared to their energies of binding in the

[1] G. F. Chew and B. L. Moyer, Am. J. Phys. **18**, 125–135 (1950), UCRL-444.

nucleus. There are also events in which the entire energy of the incident particle is taken up by the nucleus, with the production of such a large excitation as to result in the violent ejection of several nuclear particles, or in fission into two or more fragments.

II. Production of High Energy Neutrons

Since the neutron experiences no interaction with matter unless it comes within the range of the nuclear forces, it is an important projectile in the measurement of nuclear dimensions and in the study of nuclear forces by scattering methods. The strong yield of high energy neutrons in the forward direction from the target of the 184-in. cyclotron provides means of investigating nuclei and nuclear force fields with a particle beam whose wavelength is sufficiently small to delineate some general features of structure. The process by which this neutron beam is produced will first be described.

At its maximum beam orbit radius the cyclotron delivers at the target either 190-Mev deuterons or 350-Mev protons. (It will also develop 380-Mev α-particles, but these are not important in production of the neutron beams here under discussion.) The character of the neutron yield in the two cases is different, as is also the predominant process by which the high energy neutrons are produced. The case of deuteron operation will be first discussed.

A. Deuteron "Stripping."—One of the earliest observations made when the 184-in. cyclotron operations began was of the existence of an intense, relatively narrow cone of high energy neutrons projected in the direction of the motion of the deuterons as they strike the target. The intensity and angular distribution of the neutrons was not much affected by the element of which the target was composed.

Measurements[2] of the angular distribution indicated that it decreased to half intensity at 5°–6° from the axis of symmetry. The energy spectrum of the neutrons was later observed by measurements on recoil protons, both by cloud chamber techniques[3] and by proportional counter telescope methods.[4] In Fig. 1 are presented both angular and energy distribution data.

It is clear that these neutrons cannot originate in a compound nucleus of the target. Their origin is to be explained in terms of the unique properties of the deuteron; and the simple process, whose theory has been developed by Serber[5] and which accounts well for the observed distributions, is termed "stripping."

The deuteron is a loosely bound system, and its wave function indicates that its two particles actually spend considerable time apart from each other by distances greater than the "range" of nuclear forces. When a 190-Mev deuteron passes by a target nucleus the time interval of its interaction with the nucleus is smaller than the characteristic times of the deuteron's own internal motions. Consequently, it is possible for one particle of the deuteron to interact with a target nucleus and be "stripped" from the deuteron

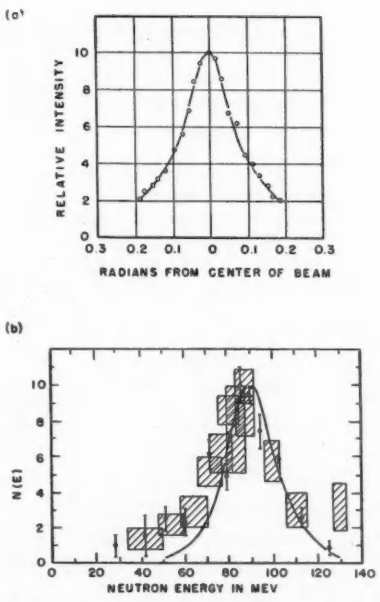

FIG. 1. (*a*) Angular distribution of neutrons produced by deuteron "stripping" at a copper target (from reference 2). (*b*) Energy distribution of neutrons from 190-Mev deuterons striking a ½-in. Be target. The shaded rectangles and the vertical lines are data from two experimental methods, and the curve is from Serber's theory (see references 4 and 5).

[2] A. C. Helmholz, E. McMillan, and D. Sewell, Phys. Rev. **72**, 1003–1007 (1947).
[3] K. Brueckner, W. Hartsough, E. Hayward, and W. Powell, Phys. Rev. **75**, 555–564 (1949).
[4] See J. Hadley, *et al.*, Phys. Rev. **75**, 351–363 (1949).
[5] R. Serber, Phys. Rev. **72**, 1008–1016 (1947).

without producing pronounced effects upon the motion of the other particle.

The energy and angular distribution of the free particle, which may be either a proton or a neutron, is then determined by its momentum at the instant of stripping. This momentum will be vectorially compounded out of the motion of the deuteron as a whole plus the relative motion of its two particles, and, because of the random phases and orientations of the latter motion, there will be a distribution in momentum of the free particle. Serber calculates that the half-width of the energy distribution should be about $\Delta E_{\frac{1}{2}}=1.5(E_d\epsilon_d)^{\frac{1}{2}}$, where E_d is the deuteron kinetic energy and ϵ_d is its binding energy. The half-angle of the angular distribution at half-intensity is $\Delta\theta_{\frac{1}{2}}=1.6(\epsilon_d/E_d)^{\frac{1}{2}}$. The reason for the appearance of the binding energy rather than the actual potential well depth is just that this particular process occurs for collisions in which the deuteron particle separation is greater than the range of forces. The greatest number of neutrons will appear with just half the original kinetic energy of the deuteron, though for a thick target this will be somewhat modified by the slowing-down of some of the deuterons before the stripping event.

The comparison of theory and observation may be seen in the curves of Fig. 1. The angular distribution for a uranium target is broader than the one displayed by about 15 percent, primarily because of the Coulomb deflection of the incident deuterons by the large nuclear charges. Similar distributions apply to protons from the target. The energy distribution of these has been studied by Chupp, Gardner, and Taylor,[6] who employed the cyclotron magnetic field as a proton spectrograph and distributed their detectors under the edge of the dee on a radial line through the target.

B. Neutrons Produced by 350-Mev Protons.—When protons of this energy collide with nuclei it is to be expected that neutrons of highest energy in the forward direction will be produced by those events in which either a "knock-on" collision of the proton with a neutron occurs, or an "exchange" collision of the type to be described in the next article of this series. In the latter case the proton in its interaction with the nuclear particles of the target is transformed into a neutron through meson exchanges which leave it without charge.

In either of these cases the net result is to replace a neutron in the nucleus by a proton, and to transform a fast-moving proton into a fast-moving neutron. The resulting neutron may, in favorable cases, possess nearly all the energy of the original proton, though it is always necessary to subtract enough energy to account for the mass difference of the initial and final nuclei.

Besides these neutrons of highest energies, there will, of course, be others from the many kinds of events which occur under such bombardment; but they will have lower energies and will not be so strongly concentrated in the forward direction.

The experimental determinations of the energy spectrum and angular distribution of this high energy neutron group are not yet complete. Tentative data show the neutron energies to extend up toward 350 Mev, with a broad maximum in the neighborhood of 270 Mev. Preliminary investigations of the angular distribution of the high energy neutrons from a Be target have been made by DeJuren, and by Wright and Miller,[7] showing a decrease to half-intensity at 25°–30° from the extended direction of the protons striking the target.

Since the nucleons with which the protons interact can have kinetic energies within the target nuclei of the order of 25 Mev, and since the exclusion principle suppresses the forward emission of nucleons (see Sec. V), it is understandable that the angular spread given to even the most energetic of the emerging neutrons will be larger than that observed for the neutrons from the stripping of deuterons.

C. Collimation of High-Energy Neutrons into a Beam.—Most of the experiments with these high energy neutrons are performed outside the 10-ft concrete shielding wall at a distance of about 60 ft from the cyclotron target. Neutrons escape through a carefully tapered hole in the shielding wall which aims at the target along the extended direction of the incident proton or deuteron beam.

[6] W. Chupp, E. Gardner, and T. B. Taylor, Phys. Rev. **73**, 742–749 (1948).

[7] Private communications.

The 90-Mev neutrons are attenuated in concrete with a reduction to half-intensity in about 9½ inches. For the 270-Mev neutrons the half-value thickness is 18 inches. Consequently the 10 ft of concrete would not produce as large a beam-to-background ration as might be desired. To improve this situation a 7-ft cube of concrete with an aligned hole is interposed between the cyclotron and the shielding wall in the direction of the neutron spray. By these means the intensity in the neutron beam defined by the collimating holes has been measured in the 90-Mev case to be a few thousand times that at a point one centimeter outside the geometrically defined beam volume. For the 270-Mev neutrons the ratio is not well known, but it is less favorable.

Flux densities of the order of 10^6 per cm² per sec are available with 90 Mev mean energy, and 10^4 per cm² per sec for 270 Mev mean energy. In Fig. 2 a plan view of the neutron beam system is shown.

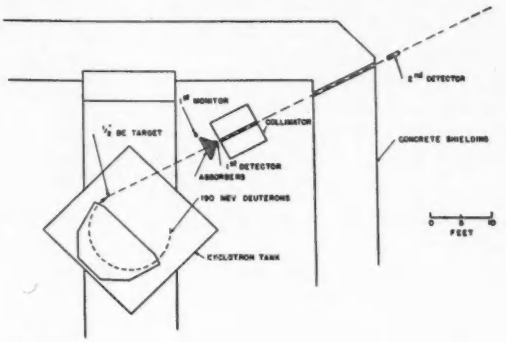

FIG. 2. Plan view of neutron beam collimating system.

III. Collisions of High Energy Neutrons with Nuclei

Before proceeding to further discussion of effects produced in nuclei of the cyclotron target by incidence of the proton or deuteron beam, a section will be devoted to description of some features of the collisions of the high energy neutrons with nuclei. Much to be said here will apply equally well to collisions of protons and deuterons with nuclei except for the effects, such as Rutherford scattering, which depend specifically upon electrical charge.

A collision is defined as any interaction of a neutron in flight with a nucleus in its path which results in changing observably the state of motion of the neutron. Collision types may be subdivided into elastic and inelastic events, and these will be described in subsequent sections. The total cross sections presented by various nuclei for a collision of any type will first be discussed.

A. Total Collision Cross Sections and Nuclear "Transparency."—The measurement of total cross section is made by observing the reduction in intensity of a directed beam of high energy neutrons by known thicknesses of matter placed between the neutron source and a detector of the neutrons in the beam. If the experiment is properly designed, any collision event which either deflects, degrades, or absorbs a neutron will result in eliminating it from possibility of actuating the detector. Correction must be made for the neutrons which are scattered by sufficiently small angles to strike the detecting volume, but good experimental arrangements make this a very small effect.

Two types of high energy neutron detectors have been employed. Cook *et al.*,[8] used small carbon disks in which neutrons of over 20-Mev energy produce the $C^{12}(n,2n)C^{11}$ reaction yielding the 20.5-minute half-life positron emission. DeJuren and Knable[9] used chambers in which pulses from fission of bismuth nuclei by neutrons of over 50-Mev energy were counted. The energy dependence of the detectors and the energy distribution in the neutron beam make the effective neutron energy in the case of carbon detectors about 84 Mev, and in the case of bismuth fission detection about 95 Mev. In Fig. 2 may be seen the arrangements of the DeJuren and Knable experiment.

The nuclear radii deduced from this experiment are plotted in Fig. 3. Deduction of radii from cross sections must be done in terms of some model of the nucleus. If an "opaque" model is assumed, i.e., a sphere which is perfectly absorbing toward the wave system describing the neutron beam, then the total cross section pre-

[8] L. Cook, E. McMillan, J. Peterson, and D. Sewell, Phys. Rev. **75**, 7 (1949).
[9] J. DeJuren and N. Knable, Phys. Rev. **77**, 606–614 (1950).

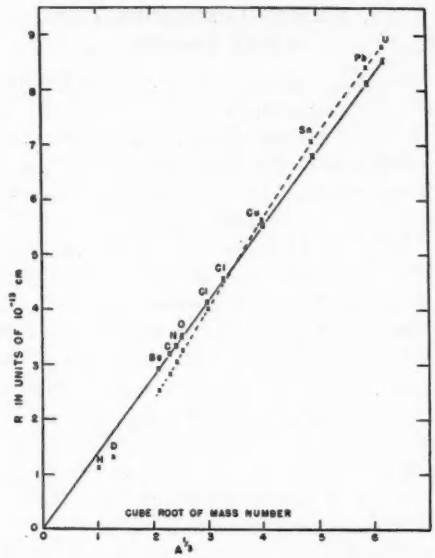

FIG. 3. Nuclear radii as deduced from measurements of total cross sections. Dashed curve arises from theory of opaque nucleus. Solid curve is from transparent nucleus theory (see reference 10).

sented by a nucleus of radius R is

$$\sigma_T = 2\pi R^2. \tag{1}$$

The factor of 2 can be related to the familiar Babinet's principle of physical optics, which leads to the conclusion that when an advancing wave disturbance meets an opaque obstacle, the total intensity of the diffracted disturbance is equal to that which is absorbed. Thus, the cross section for elastic scattering of the neutrons, which will be represented by diffraction of their wave system, will be equal to the cross section for "absorption," meaning any interaction other than elastic scattering. But, since at this neutron energy the geometrical area presented by the nucleus must define the absorption cross section, it follows that the total collision cross section will be twice this value.

Since the density of nuclear matter is thought to be constant, it is to be expected on the basis of an opaque model of the nucleus that total cross sections would be proportional to $A^{\frac{2}{3}}$, where A is the atomic number, and that nuclear radii deduced by relationship (1) would be proportional to $A^{\frac{1}{3}}$. The plot of R *vs* $A^{\frac{1}{3}}$ in Fig. 3 indicates the way in which experimental results deviate from the opaque model predictions (see dashed line with crosses). The results for large A approach the above description, showing a tendency to become proportional to $A^{\frac{1}{3}}$; but for the nuclei below $A \simeq 60$ the simple opaque model theory is not an accurate representation.

An interesting interpretation of this discrepancy is provided by the "transparent" nuclear model of Fernbach, Serber, and Taylor.[10] The volume of space occupied by the nucleus is considered to possess an "absorption coefficient" for the neutron wave. From a particulate point of view this is equivalent to saying that a high energy neutron will have a certain mean free path in nuclear matter with respect to collisions with the constituent nucleons.

Not only will nuclear matter possess an absorption coefficient, but it will also display an "index of refraction" for the neutron wave, by which phase displacements for the penetrating wave relative to the unperturbed wave will occur. This effect is, of course, due to the change in potential of the field in which the neutron moves as it enters or leaves the nuclear volume.

By selecting appropriate values for these two parameters, and then using the mathematical relationships of the transparent model theory to calculate nuclear radii from the observed values of σ_T, the close proportionality of R to $A^{\frac{1}{3}}$ shown in the indicated curve of Fig. 3 is obtained. The absorption coefficient employed for the case of 84-Mev neutrons was 2.2×10^{12} cm^{-1}, corresponding to a mean free path of 4.5×10^{-13} cm. For the 95-Mev case which is shown in Fig. 3 these values were 3.0×10^{12} cm^{-1}, and 3.3×10^{-13} cm.

The value of σ_T for neutrons of 270-Mev mean energy have not yet been published. Measurements indicate that they are approximately 0.6 of the values at 95 Mev. Transparency effects are presumably much more pronounced at this energy.

B. Elastic Scattering of Neutrons at 84 Mev.— It was mentioned above that in the opaque nuclear model the cross section for elastic scattering should be one-half the total collision cross section. Moreover, the angular distribution of the elastically scattered neutrons should be

[10] S. Fernbach, R. Serber, and T. B. Taylor, Phys. Rev. **75**, 1352–1355 (1949).

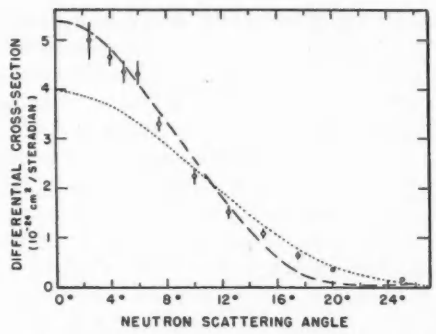

FIG. 4. Angular distribution of neutrons elastically scattered from aluminum nuclei. The dashed curve is the distribution predicted by the opaque nucleus theory, and the dotted curve by the transparent nucleus theory.

described by the Fraunhofer diffraction pattern of a plane wave encountering a perfectly absorbing sphere. This is approximately the same as the Fraunhofer pattern for a circular disk, which is

$$I(\theta) \sim [J_1(KR \sin\theta)/\sin\theta]^2,$$

where J_1 is the first-order Bessel Function, and $K = 1/\lambda$ is the propagation constant for the incident wave system.

A study[11] of the angular distribution has given results similar to the central maximum of the Fraunhofer pattern, but departing somewhat in a manner which is qualitatively expected from the transparent model. Figure 4 illustrates the comparison with opaque model predictions.

By integration of the differential scattering curves a value for the elastic scattering cross section is obtained which is always slightly greater than $0.5\sigma_T$. This also is predicted by the transparent model.

Complementary measurements of the cross sections for inelastic collisions have yielded values always a few percent less than $0.5\sigma_T$. These measurements are made in the broad neutron beam ahead of the collimator system and are so designed that elastic scattering events do not remove neutrons from detectability. The reduction of detected neutrons is then solely due to events which absorb the neutrons or degrade their energy below the detection threshold.

[11] A. Bratenahl, S. Fernbach, R. H. Hildebrand, C. E. Leith, and B. J. Moyer, Phys. Rev. **77**, 597–605 (1950).

IV. Nuclear Disintegration Types at High Energies

The effects produced when the cyclotron projectiles or the high energy neutrons enter target nuclei include a great variety of events. They will here be discussed under such classifications as "spallation," fission, and knock-on collisions with constituent nuclear particles.

A. Nuclear Chemistry; Spallation and Stars.—Chemical identification of radioactive reaction products has shown results which are frequently difficult to classify by stating the incident particle and the expelled particles in the usual form, e.g., (p, α), etc.; for the emitted particles may include in total several protons and neutrons; and it is not possible by chemical means to judge whether the emitted nucleons came away singly or in units such as H^2, H^3, He^3, He^4, etc. Emission of fragments as large as Li^8 has been identified by film track and cloud chamber studies.

For the purposes of indicating such reactions, Seaborg, Perlman, and their associates have adopted a symbolism such as

$$_{33}As^{75}(d, 9p12n)_{25}Mn^{56},$$

which is understood to enumerate the ejected nucleons but not necessarily show the form in which they are emitted. Such reactions have been termed "nuclear spallation."

For examples of spallation effects, reference can be made to the study of Miller, Thompson, and Cunningham[12] who found from the bombardment of Cu with 190-Mev deuterons radioactive products distributed from $_{15}P^{32}$ to $_{30}Zn^{63}$. A striking array of products was observed by O'Connor and Seaborg[13] from the bombardment of U^{238} with 380-Mev alpha-particles. Through the processes of fission and spallation a continuous distribution of products was produced extending down to $Z = 25$ (Mn). It is estimated that mass numbers down to about 180 are due to spallation, and those lower to fission.

The discovery[14] of the delayed emission of neutrons from production of N^{17} provided

[12] D. R. Miller, R. C. Thompson, and B. B. Cunningham, Phys. Rev. **74**, 347–348 (1948).
[13] P. R. O'Connor and G. T. Seaborg, Phys. Rev. **74**, 1189–1190 (1948).
[14] N. Knable, E. O. Lawrence, C. E. Leith, B. J. Moyer, and R. L. Thornton, Phys. Rev. **74**, 1217 (1948).

a simple means of studying the yields of spallation reactions leading to N^{17} from elements lying above it in atomic number. Chupp and McMillan[15] have studied the relative yields of N^{17} from various elements and isotopes under bombardment by 190-Mev deuterons extending as far as sulfur and were able to show systematic effects such as augmented yields for those cases where emission of alpha-particles by the compound nucleus could lead to N^{17}.

Graphic portrayals of nuclear spallation may be seen in the cloud chamber photographs of nuclear "stars" produced by the high energy neutrons. Figure 5 shows one such photograph among many obtained by W. M. Powell[16] and his associates. Figure 6 shows a nuclear star in photographic emulsion in which one of the ejected fragments is a Li^8 nucleus, which subsequently decays (half-life = 0.88 sec) into a Be^8 nucleus, which immediately divides into two alpha-particles.

Systematic studies of the number, type, and distribution of star prongs have been made by Gardner and Peterson,[17] from which some information may be inferred about the excitation energy of the bombarded nuclei. Ejected neutrons, of course, leave no tracks, and estimates of their number and energies must be attempted.

B. High Energy Fission.—Fission by high energy bombardment displays notable differences from the familiar slow neutron fission. Kelly and Wiegand[18] identified fission by the 90-Mev neutron beam in elements ranging from Pt to Bi and measured the relative fission cross sections. They determined the threshold for fission of Bi by neutrons to be approximately 50 Mev, and this knowledge has served since as a convenient means of detecting neutrons of greater energies than 50 Mev (see Sec. III-A).

The mass distribution of fission fragments in high energy fission is strikingly different from that in the slow neutron case. O'Connor and Seaborg[13], and Goeckermann and Perlman[19] have made extensive chemical identifications of high energy fission products from U and Bi. The most probable partition of the parent nucleus is into two equal fragments, in contrast to the asymmetric character familiar with slow fission. Moreover, the mass number of these most probable fragments is not one-half the mass number of the original bombarded nucleus, but distinctly less than this value, whereas the atomic number of these fragments is one-half that of the bombarded nucleus. This suggests that the parent nucleus in the fission event is a neutron-deficient isotope of the bombarded element, formed by evaporating approximately 12 neutrons from the bombarded nucleus before fission occurs.

Another notable feature is that the n-p ratio in the fragments is the same as that of the parent nucleus. This means that when fission into fragments of unequal mass occurs, the lighter fragment will possess an excess of neutrons and will most often be β^--active, whereas the heavier fragment will be deficient in neutrons and will usually be β^+-active. This again is in contrast to the slow fission situation for which all fragments have typically an excess of neutrons. The infer-

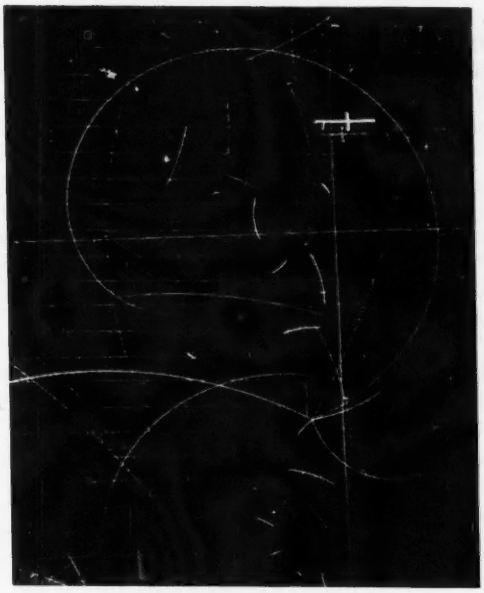

FIG. 5. Cloud chamber photograph of nuclear star induced by high-energy neutron. Magnetic field aids in identifying particles by measurement of $B\rho$. (Photograph from W. M. Powell.)

[15] To be published.
[16] W. M. Powell, Phys. Rev. **72**, 739 (1947).
[17] E. Gardner and V. Peterson, Phys. Rev. **75**, 346–369 (1949); E. Gardner, Phys. Rev. **75**, 379–382 (1949).
[18] E. Kelly and C. Wiegand, Phys. Rev. **73**, 1135–1139 (1948).
[19] R. H. Goeckermann and I. Perlman, Phys. Rev. **73**, 1127–1128 (1948).

ence from this is that the fission process is fast in the high energy case and that the parent nucleus is highly excited even after considerable energy has been dissipated in the preceding "boiling-off" of 10 to 12 neutrons.

Fission of thorium induced by 40-Mev alpha-particles from the 60-in. Crocker Laboratory cyclotron has been studied by A. S. Newton.[20] Here the partition of the nucleus displays the characteristic asymmetry in the relative yields of products; but the ratio of peak to valley yields is only about 2, as compared to 600 for slow neutron fission of uranium. The processes characteristic of high energy fission are in evidence here, though the excitation of the parent nucleus is not so high as to prevent evidence of some rearrangement into the energetically favorable distributions which give the double hump.

C. Ejection of Fast Particles; Subnuclear Collisions. 1. Emission of Deuterons and Tritons.— Upon beginning a study with proportional counters of the emission of high-energy protons from nuclei under bombardment by the 90-Mev neutron beam, York and Hadley[21] discovered an unexpected yield of energetic deuterons strongly concentrated in the forward direction. The same observation was made almost simultaneously in cloud chamber studies by Brueckner and Powell,[22] and subsequently by Bradner[23] using film track methods.

The reason for this being unexpected lies in the fact that the deuteron is such a loosely bound system. In order that a deuteron may emerge, it would be necessary for a proton and a neutron in the nucleus to receive relative momenta appropriate to those in a deuteron, together with

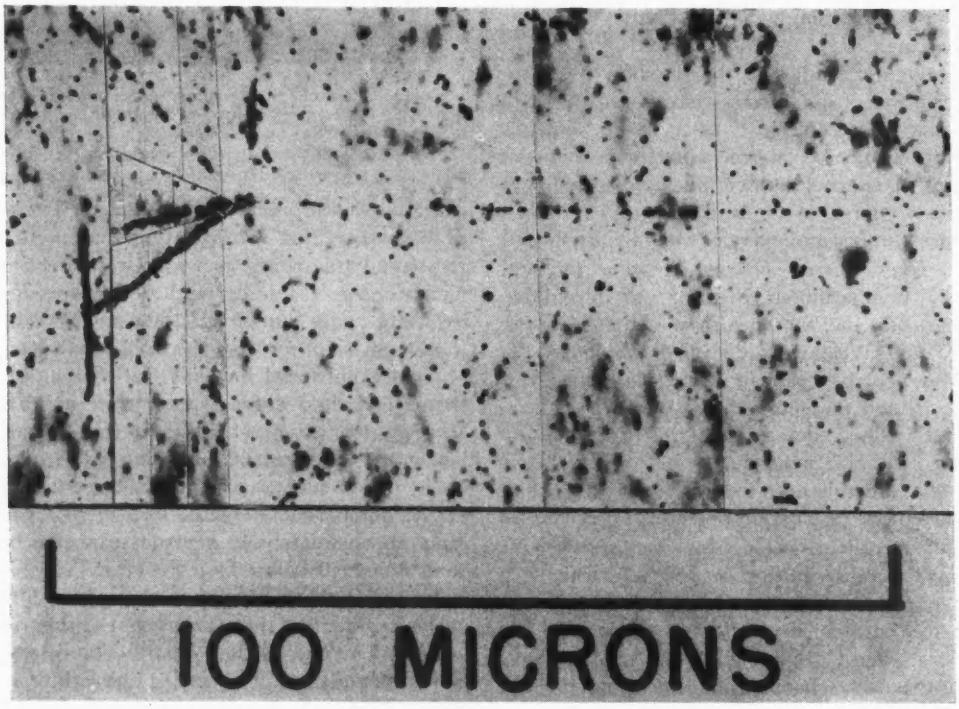

FIG. 6. Neutron-induced nuclear star in photographic emulsion showing ejection of a Li^8 nucleus. The heavy track projected downward and to the left is the Li^8, which subsequently disintegrates into two alpha-particles moving in opposite directions.

[20] A. S. Newton, Phys. Rev. **75**, 17–29 (1949).
[21] H. York, Phys. Rev. **75**, 1467 (1949); J. Hadley and H. York, to be published in Phys. Rev., UCRL-359 Revised.
[22] K. Brueckner and W. M. Powell, Phys. Rev. **75**, 1274 (1949).
[23] H. Bradner, Phys. Rev. **75**, 1467 (1949).

a total momentum sufficiently great to give a fair probability of penetrating through nuclear matter to the surface without further collision. The likelihood of accomplishing this by nuclear excitation or by knock-on collisions with subunits of the nuclear system seems at first thought remote.

Further light is shed upon the mechanism by the fact that the angular distribution is strongly peaked forward, and by the observation that the deuterons emerging directly forward have an energy distribution with a peak at 60–65 Mev and a width at half intensity of about 25 Mev. It is clear that the mechanism will not involve a compound nucleus, but rather a subnuclear interaction between the bombarding neutron and a constituent proton.

Chew and Goldberger[24] were able to give quite a satisfactory explanation of a mechanism by which these deuterons are produced. The nucleons within a nucleus are in motion, and their kinetic energies will extend as high as about 25 Mev. Now when a bombarding neutron penetrates through the nucleus there is a significant probability that a proton within the nucleus can be moving with direction and speed so related to the neutron as to correspond to a state of motion of a deuteron. The two particles then continue together, and if the subsequent path within the nucleus is not large they can emerge intact as a deuteron.

It is now clear why these deuterons are strongly peaked in the forward direction and show a rather high average energy. Only those protons with large momenta parallel to the neutron direction will be captured. Consequently, a deuteron thus formed can be moving only within a very narrow angular range about the direction of the neutron, and its momentum compounded out of that of the neutron and proton will forbid a low kinetic energy.

A numerical example will illustrate the point. In Fig. 7 are pictured a 100-Mev neutron and a 25-Mev proton; the former represents a bombarding neutron moving through a nucleus, and the latter one of the nuclear protons. The neutron momentum is (nonrelativistically) $p_n=(2mE_n)^{\frac{1}{2}}=10f(2m)^{\frac{1}{2}}$, since $E_n=100$ Mev, and f is a factor

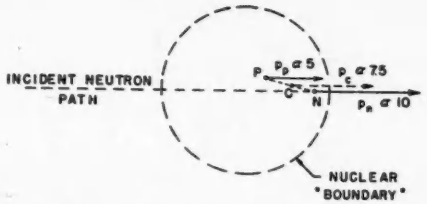

FIG. 7. Representation of formation of deuteron by neutron-proton "pick-up" within nucleus. Point C is the center of mass for a high-energy neutron N and a nuclear proton P. The vectors represent momenta.

to adjust the units. The proton momentum is $p_p\sim 5f(2m)^{\frac{1}{2}}$. The momentum of each relative to their center of mass is $2.5f(2m)^{\frac{1}{2}}$, so that the kinetic energy of each relative to the center of mass is only 6.25 Mev. So from the standpoint of internal kinetic energy, the combination of these two particles to form a deuteron need not be forbidden.

If their relative orbits were such as to agree with an allowed state of the deuteron, its momentum would be

$$p_d=p_n+p_p=15f(2m)^{\frac{1}{2}}=[2(2m)E_d]^{\frac{1}{2}}.$$

Its kinetic energy E_d would then be 112 Mev, and upon emerging from the nuclear potential well its observed energy may be about 95 Mev.

York's measurements indicate that about 1/12 of all inelastic collisions of 90-Mev neutrons with carbon nuclei result in deuteron ejection. For lead nuclei the fraction is approximately 1/25.

Besides deuterons, a much smaller number of tritons is also observed in the forward direction with energies between 35 and 70 Mev. Some such "pick-up" process must also operate to produce these, in which the incident nucleon and *two* of the target nucleons just happen to have momenta relative to each other which are compatible with the internal motion of a triton.

2. Knocking-Out of Protons.—The Hadley-York experiment,[21] and the cloud chamber observations,[22] on the protons emerging from struck nuclei demonstrate energies extending up to well over 100 Mev and angular distributions concentrated so strongly forward as to lead to the conviction that they arise predominantly from n-p collisions within the nuclei. This at least can be stated for those protons emerging with energies in excess of 20 Mev.

For protons knocked out of carbon into differ-

[24] G. F. Chew and M. L. Goldberger, Phys. Rev. 77, 470–475 (1950).

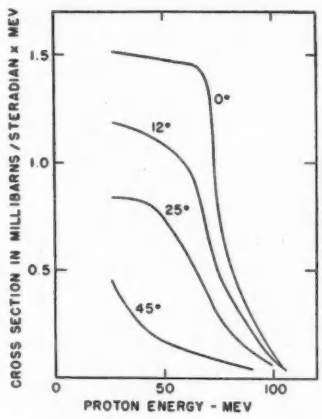

FIG. 8. Energy spectra for protons knocked out of carbon nuclei at various angles by 90 Mev neutrons (see reference 21 for details and probable errors).

entials of solid angle at various angles the energy spectra are portrayed in Fig. 8. York finds that, in 0.4 of the cases of inelastic collision of 90-Mev neutrons with carbon nuclei, protons of energy greater than 20 Mev emerge. For copper and lead these fractions are 0.31 and 0.24, respectively. Cloud chamber pictures also demonstrate that of the energy dissipated in matter by the passage of high energy neutrons, a large fraction is represented in the production of energetic protons. This is of significance in estimating the biological effects of such neutrons.

Though these protons originate in *n-p* collisions within the nucleus, it must not be presumed that the energy and angle distributions are representative of free *n-p* scattering. For a proton within the nucleus is not at rest, but has momentum which must be vectorially compounded with that contributed by the neutron in the collision. In addition the Pauli principle acts to prevent small momentum transfers, as will be described in the following section. Also there may have been prior collisions of the neutron in the nucleus, and subsequent collisions of the proton before it emerges.

V. Dynamics of Nuclear Collisions at High Energies

In a brief article published[25] in the early days of the operation of the 184-in. cyclotron, Serber outlined some outstanding features to be expected of collisions of high energy nuclear projectiles with nuclei. Many of these features have been exemplified by experiments described in this paper. A condensed summary of the physical ideas involved is now presented.

1. The interaction time for a collision between the projectile and a nucleon is small compared to the time between collisions of particles within the nucleus. Hence the nucleus as a system does not "know" of the collision until after it has occurred. This same feature was previously presented in terms of the ratio of nuclear size to the wavelength for the projectile beam.

2. Since the collision cross section between nucleons decreases with relative energy, it is to be expected that nuclei will demonstrate transparency for high energy particles. For 90-Mev neutrons the mean free path in nuclei has been seen to be about 4×10^{-13} cm.

3. The momentum transfers to struck nucleons are not usually large compared to characteristic momenta of nucleons in nuclei. Hence the observed angular and energy distributions will not be described by free-particle collisions.

4. Very small momentum transfers are suppressed by the exclusion principle on these grounds: if nuclear matter is represented by a Fermi gas bounded by the nuclear volume, then all momentum states are occupied up to a certain maximum value. Thus, only those collisions are allowed which will transfer sufficient momentum to lead to an unoccupied state.

5. In large nuclei the incident and struck nucleons will usually have further collisions before emerging. The energy is thus distributed over the nucleus, resulting in excitation which is relieved by the boiling-off of many particles and producing the spallation reactions.

6. In light nuclei the energy delivered will be largely carried off by relatively few particles, such as knocked-out protons and "pick-up" deuterons.

In the next paper of this series the discussion will concern free-particle scattering experiments at high energy and their bearing upon nuclear forces. This work was performed under the auspices of the Atomic Energy Commission.

[25] R. Serber, Phys. Rev. **72**, 1114–1115 (1947).

Boner

"The Quantum Theory states that light is *particular* in nature."—From a Haverford examination.

Enrollments and Degrees Awarded to Physics Majors

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Data are presented showing the enrollments of undergraduate and graduate students in physics in approximately 500 colleges and universities in the United States. In the academic year 1949–50, 12,670 undergraduate and 5,560 graduate physics majors were enrolled. These numbers are about twice those of four years previous. During 1948–49 the number of degrees in physics granted were: bachelors, 3,495; masters, 920; doctors, 274. For 1949–50 the number of degrees to be granted was estimated as: bachelors, 4050; masters, 1,000; doctors, 340. The production of doctorates in physics has now reached a "normal" output, based on previous trends which indicated a rapid increase. A tabulation is presented of the bachelors, masters, and doctors degrees granted in 1948–49 by each of the 500 colleges and universities in the United States.

STUDENTS enrolled as physics majors and the numbers of students awarded degrees in this science are continuing at an extraordinarily high level, although there seems to be evidence that a peak in enrollment has been reached. Data on the production of doctorates in physics have been kept for many years.[1] But statistical surveys of the number of physicists in training at the bachelor's and master's levels have been made only in recent times.[2] Little effort has been expended in attempting to analyze and predict the occupational outlook for physicists,[3] although a study of this sort is now being started by the Bureau of Labor Statistics of the United States Department of Labor.

TABLE I. Enrollment of physics majors.

Year	Undergraduates	Graduate students
1946–47	8760	2640
1947–48	10,308	3999
1948–49	11,700	4850
1949–50	12,670	5560

In 1949 there were about 1800 colleges, universities, and other institutions of higher learning in the United States which offered at least a two-year program. Of these about 500 were junior colleges, 146 were primarily for teachers, and the remaining 1154 included some 660 conventional liberal arts colleges, 250 technical and professional institutions, and 250 other colleges and universities. Approximately 500 of these institutions report themselves as offering an undergraduate physics major. This was arbitrarily defined as one including a minimum of 22 semester hours in physics, of which at least eight require the calculus as a prerequisite. Over 140 institutions give master's degrees in physics and some 68 grant the doctorate.

Beginning with the academic year 1946–47, the writer has made an annual survey of physics students through the facilities of Sigma Pi Sigma, physics honor society. A postal card questionnaire was sent to the head of each physics department and two followup requests were made to those not replying to the first inquiry. Usable results have been obtained from 95 percent of those queried. Grateful acknowledgment is made to these busy administrators for their cooperation in providing the required data.

The total enrollment of physics majors is

TABLE II. Degrees granted to physics majors.

	Bachelor's			(Estimate)	Master's			(Estimate)	Doctor's			(Estimate)
	46–47	47–48	48–49	49–50	46–47	47–48	48–49	49–50	46–47	47–48	48–49	49–50
Degrees granted	1570	2400	3495	4050	499	825	920	1000	105	228	274	340
No. of institutions	372	376	519	520	131	137	142	146	66	68	64	66
Av. no. per institution	4.2	6.4	6.8	7.8	3.8	6.1	6.5	6.9	1.6	3.4	4.3	5.2

[1] M. H. Trytten, Sci. Monthly **50**, 37–47 (1945).
[2] M. W. White, Rev. Sci. Instr. **18**, 219–222 (1947).
[3] M. W. White, Phys. Today **2**, No. 11, 16–17 (1949).

TABLE III. Doctorates granted in physics, September 1, 1948–August 31, 1949.

Massachusetts Institute of Technology	37	Duke	4
Harvard	20	Minnesota	4
Yale	14	Pennsylvania State College	4
California Institute of Technology	12	Rice	4
California (Berkeley 7) (Los Angeles 4)	11	North Carolina	3
		Oklahoma	3
		Pennsylvania	3
Indiana	11	Case	2
Chicago	10	Cincinnati	2
Illinois	10	Iowa State	2
Johns Hopkins	10	Missouri	2
Rochester	10	Notre Dame	2
Wisconsin	9	Pittsburgh	2
Carnegie Institute of Technology	8	Texas	2
		Boston University	1
		Catholic	1
Princeton	8	Colorado	1
Brown	7	George Washington	1
Cornell	7	Illinois Institute of Technology	1
Ohio State	7		
Northwestern	6	Maryland	1
Purdue	6	New York	1
Virginia	6	Oregon State	1
Columbia	5	Radcliffe	1
Michigan	5	St. Louis	1
Washington (St. Louis)	5	Stanford	1

TABLE IV. Degrees conferred July 1, 1948–June 30, 1949.

Field	Bachelor's	Master's	Doctor's
All (totals)	366,634	50,827	5293
Astronomy	20	10	11
Bacteriology	780	293	64
Biochemistry	175	123	84
Biology	8275	404	103
Botany	424	192	78
Chemistry	9132	1427	749
Education	37,765	13,828	681
Engineering (total)	43,604	4647	360
Aeronautical	1935	342	11
Chemical	4206	680	125
Civil	6299	734	28
Electrical	11,042	991	65
Mechanical	12,277	823	41
Miscellaneous	7845	1077	90
Geology	1851	385	88
Mathematics	5040	893	126
Metallurgy	357	123	51
Miscellaneous sciences	7768	714	39
PHYSICS	2828	841	266
Psychology	8205	1455	201
Zoology	2835	436	78

shown in Table I. It is well known that the enrollments prior to World War II were much smaller than those of recent years. With the general decline of students in institutions of higher learning it is to be expected that the number of physics majors should level off or even decrease. The numbers of students at the graduate level, however, is likely to continue to increase for several years as the bulge in the enrollment curve reaches the more advanced periods of training.

The numbers of degrees granted to physics majors in recent years, together with the numbers estimated for the present academic year, are given in Table II. The years indicated are for

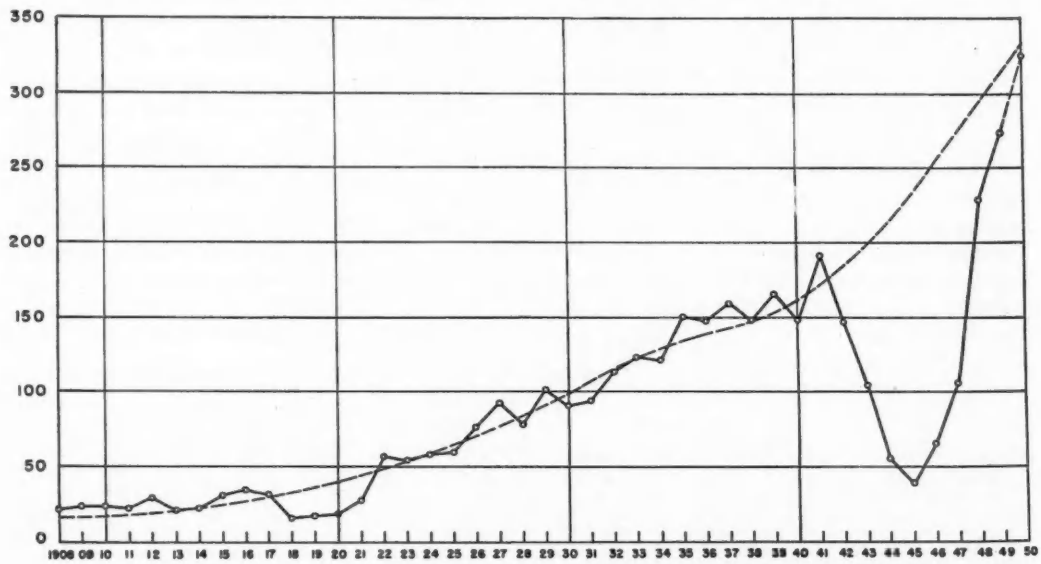

FIG. 1. Number of doctoral degrees in physics granted in the United States since 1912.

TABLE V. Degrees granted in physics 1948–49. Institutions designated with an asterisk granted no undergraduate degrees to students having a physics major during the year 1948–1949, although it is expected that students will be graduated with majors in physics in the near future. The double asterisk designates institutions believed to offer a physics major, for which no data concerning graduates during the year 1948–1949 were obtained. Columbia University offers no undergraduate major in physics.

	Bachelor's	Master's	Doctor's
Adelphi	6		
Adrian	0		
Agnes Scott	3		
Agricultural & Mechanical College (Arkansas)	0		
Agricultural & Mechanical College (Texas)	12	2	
Agricultural & Technical College (North Carolina)	0		
Akron	3	5	
Alabama Polytechnic Institute	9	5	
Alabama	7	1	
Alaska	3		
Albany State (Georgia)	**		
Albion	2		
Alfred	1		
Allegheny	5		
Alma	1		
American International	3		
American	4		
Amherst	5	0	
Antioch	4		
Arizona State	0		
Arizona	5	1	
Arkansas State Teachers College (Conway)	**		
Arkansas	6	3	
Atlantic Union	3		
Augustana (Illinois)	0		
Augustana (South Dakota)	6		
Aurora	1		
Austin	1		
Baker	2		
Baldwin-Wallace	5		
Ball State Teachers College (Indiana)	**		
Barat	1		
Bard	0		
Barnard	2		
Bates	9		
Baylor	8		
Beloit	10		
Bennington	0		
Berea	6		
Berry	0		
Bethany	5		
Bethel (Kansas)	0		
Birmingham-Southern	2		
Boston	44	6	
Boston University	11	16	1
Bowdoin	8		
Bowling Green State	3		
Bradley	3	0	
Brigham Young	9	0	
Brooklyn	30		
Brown	15	10	7
Bryn Mawr	2	2	
Bucknell	8	4	
Buena Vista	0		
Buffalo	12	0	0
Butler	4		
California Institute of Technology	18	15	12
California	139	42	11
Berkeley	(68)	(9)	(7)
Los Angeles	(71)	(33)	(4)
Canisius	10		
Capital	2		
Carleton	5		
Carnegie	30	10	8
Carroll (Wisconsin)	3		
Carson-Newman	4		
Carthage	**		
Case Institute of Technology	35	12	2
Catawba	5		
Catholic University	11	7	1
Centenary	0		
Central (Iowa)	1		
Central (Missouri)	0		
Central Michigan	8		
Central Missouri	10		
Central State (Oklahoma)	5		
Central Washington	0		
Centre (Kentucky)	3		
Chadron State Teachers College (Nebraska)	0		
Chapman	0		
Chattanooga	5		
Chestnut Hill	1		
Chicago	16	25	10
Cincinnati	4	7	2
The Citadel	5		
Clark & Morehouse	1		
Clark	7		
Clemson	8	2	
Coe	3		
Colby	5		
Colgate	3		
Colorado Agricultural & Mechanical	3		
Colorado College	3		
Colorado	23	3	1
Columbia		55	5
Concord	*		
Concordia (Minnesota)	6		
Connecticut College for Women	3		
Connecticut	7		
Cornell (Iowa)	0		
Cornell (New York)	8	9	7
Creighton	2		
Culver-Stockton	3		
Dartmouth	6	3	
Davidson	8		
Davis & Elkins	1		
Dayton	4		
Defiance	0		
Delaware	5	2	
Delta State Teachers College (Mississippi)	1		
Denison	10		
Denver	14	4	
De Paul	10	0	
DePauw	6		
Detroit	4	4	
Dickinson	4		
Doane	2		
Drake	6		
Drew	2		
Drury	4		
Dubuque	0		
Duke	7	3	4

TABLE V.—(*Continued*).

	Bachelor's	Master's	Doctor's
Duquesne	3		
Earlham	6		
East Central State Teachers College (Oklahoma)	5		
East Tennessee State Teachers College	**		
East Texas State Teachers College	8	2	
Eastern Illinois State	6		
Eastern Kentucky State Teachers College	2		
Eastern New Mexico University	1		
Eastern Washington College	**		
Elon	2		
Emmanuel Missionary	4		
Emory & Henry	**		
Emory	7	9	
Evansville	10		
Fairmont State (West Virginia)	4		
Fenn	3		
Fisk	8		
Florida Southern	2		
Florida State (Tallahassee)	14	2	
Florida (Gainesville)	1	3	
Fordham	25	15	0
Fort Hays Kansas State	2	1	
Franklin	2		
Franklin & Marshall	6	1	
Fresno State	6	0	
Friends	2		
Furman	**		
Geneva	1		
George Washington	10	5	1
Georgetown (Kentucky)	0		
Georgetown	3	1	
Georgia Institute of Technology	15	1	
Georgia	8	3	
Gettysburg	19		
Gonzaga	*		
Goshen	1		
Goucher	3		
Greenville	4		
Grinnell	5		
Grove City	4		
Guilford	4		
Gustavus Adolphus	4		
Hamilton	1		
Hamline	4		
Hampden-Sydney	2		
Hampton Institute	0		
Hanover	6		
Hardin-Simmons	0		
Hartwick	3		
Harvard	97	29	20
Hastings	12		
Haverford	4		
Hawaii	1	2	
Heidelberg	2		
Henderson State Teachers College (Arkansas)	0		
Hillsdale	6		
Hiram	6		
Hobart & William Smith	5		
Hofstra	12		
Hollins	3		
Holy Cross	8		
Hope	2		
Houghton	2		
Houston	8		
Howard	1		
Howard University	3	3	
Hunter	2		
Huntington (Indiana)	0		
Idaho, College of	0		
Idaho State	3		
Idaho	4	1	
Illinois	2		
Illinois Institute of Technology	7	7	1
Illinois	29	29	10
Illinois Wesleyan	2		
Incarnate Word	**		
Indiana Central	**		
Indiana State Teachers College (Terre Haute)	10		
Indiana	24	9	11
Iona	6		
Iowa State (Ames)	5	1	2
Iowa State Teachers College (Cedar Falls)	3		
Iowa State University	11	7	0
Iowa Wesleyan	2		
Jamestown	**		
John Carroll	12	*	
Johns Hopkins	7	1	10
Johnson C. Smith	**		
Juniata	2		
Kalamazoo	8		
Kansas City	3		
Kansas State	19	4	
Kansas State Teachers College (Emporia)	10	2	
Kansas State Teachers College (Pittsburg)	**		
Kansas	20		0
Kent State	5		
Kentucky	11	6	
Kenyon	4		
King	0		
Knox	5		
Lafayette	10		
Lake Forest	2		
LaSalle	3		
La Sierra	3		
Lawrence	7		
Lebanon Valley	**		
Lehigh	24	13	
Lewis & Clark	10		
Lincoln Memorial	4		
Lincoln (Missouri)	2		
Lincoln (Pennsylvania)	**		
Linfield	4		
Long Island	4		
Louisiana College	**		
Louisiana Polytechnic Institute	1		
Louisiana State	12	6	0
Louisville Municipal College	0		
Louisville	13		
Loyola (Maryland)	12		
Loyola (Illinois)	1		
Loyola (Louisiana)	7		
Luther (Iowa)	2		
Lynchburg	11		
Macalester	4		
Madison (Virginia)	**		
Maine	26	0	
Manchester	7		
Manhattan	5		
Manhattanville	0		

TABLE V.—(*Continued*).

	Bachelor's	Master's	Doctor's
Mankato State Teachers College (Minnesota)	4		
Marietta	3		
Marquette	5	3	
Marshall	5		
Mary Manse	*		
Maryland	10	12	1
Maryville (Missouri)	*		
Maryville (Tennessee)	1		
Massachusetts Institute of Technology	47	12	37
Massachusetts	7		
Mercer	7		
Miami (Florida)	7		
Miami (Ohio)	13	0	
Michigan College of Mining & Technology (Houghton)	7		
Michigan College of Mining & Technology (Sault Ste. Marie)	**		
Michigan State	17	4	0
Michigan	72	48	5
Middlebury	11		
Millsaps	8		
Milton	2		
Milwaukee-Downer	1		
Minnesota (Duluth Branch)	8		
Minnesota (Minneapolis)	16	26	4
Misericordia	0		
Mississippi	8		
Mississippi State	0		
Mississippi	4	2	
Missouri School of Mines	2	3	
Missouri	6	5	2
Missouri Valley	1		
Monmouth	4		
Montana State College	14	1	
Montana State University	3	1	
Moravian College (Men)	6		
Morehead State (Kentucky)	2		
Morningside	4		
Mt. Holyoke	4	0	
Mt. Mercy	**		
Mt. Union	4		
Muhlenberg	12		
Mundelein	**		
Murray State Teachers College (Kentucky)	6		
Muskingum	4		
Nebraska	7	1	
Nebraska Wesleyan	5		
Nevada	2		
New Hampshire	2		
New Jersey College for Women	3		
New Mexico College of Agriculture & Mechanic Arts	4		
New Mexico Highlands	*		
New Mexico Military Institute	*		
New Mexico School of Mines	0		
State University of New Mexico	4	4	0
New Rochelle	0		
New York City College	50		
New York State Teachers College (Albany)	*	1	
New York University	36	29	1
University Heights	(12)	(5)	(0)
Washington Square College	(24)	(24)	(1)
Newcomb	**		
North Carolina College (Durham)	1		
North Carolina	10	9	3
North Carolina State College of Agriculture and Mechanics	*		
North Central (Illinois)	4		
North Dakota State	0		
North Dakota	2		
North Georgia	13		
North Texas State Teachers College	6	0	
Northeast Missouri State Teachers College	0		
Northeastern	14		
Northern Illinois State Teachers College	2		
Northern Michigan College of Education	0		
Northern New Mexico Normal	**		
Northwest Missouri State Teachers College	4		
Northwestern State (Louisiana)	2		
Northwestern State (Oklahoma)	5		
Northwestern	25	15	6
Notre Dame	3	10	2
Oberlin	12	1	
Occidental	3	1	
Ohio Northern	**		
Ohio State	24	23	7
Ohio University	5	2	
Ohio Wesleyan	6		
Oklahoma Agricultural & Mechanical	12	9	
Oklahoma Baptist	2		
Oklahoma City	3		
Oklahoma	28	12	3
Olivet	1		
Oregon State	20	9	1
Oregon	4	6	
Oshkosh State Teachers College (Wisconsin)	2		
Ottawa	3		
Otterbein	2		
College of Pacific	0		
Pacific Lutheran	0		
Pacific Union	3		
Pacific	2		
Park	0		
Parsons	0		
Pennsylvania State	26	10	4
Pennsylvania	5	15	3
Philander Smith	0		
Phillips	4		
Pittsburgh	21	2	2
Platteville State Teachers College (Wisconsin)	3		
Polytechnic Institute of Brooklyn	11	7	0
Pomona	8		
Portland	16		
Prairie View Agricultural & Mechanical (Texas)	**		
Presbyterian College of South Carolina	1		
Princeton	8	9	8
Principia	6		
Providence	7		
Puerto Rico College of Agricultural and Mechanical Arts	**		
Puerto Rico	**		
College of Puget Sound	3		
Purdue	30	14	6
Queens	6		

TABLE V.—(*Continued*).

	Bachelor's	Master's	Doctors'
Radcliffe	3	3	1
Randolph-Macon	**		
Randolph-Macon Women's College	2		
Redlands	8	1	
Reed	12		
Rensselaer	30	2	0
Rhode Island State	8	2	
Rice Institute	11	10	4
Richmond	19		
Ricker College	*		
Rick's (Indiana)	**		
Ripon	3		
River Falls State Teachers College (Wisconsin)	0		
Roanoke	9		
Rochester	13	4	10
Rockford	0		
Rockhurst	3		
Rollins	2		
Roosevelt	6		
Rutgers	6	2	0
Newark College of Rutgers	1		
St. Ambrose (Iowa)	0		
St. Bernardine of Siene	19		
St. Bonaventure	4		
St. Edward's	*		
St. Francis (Pennsylvania)	7		
St. John's (New York)	1		
St. John's (Minnesota)	3		
St. Joseph's (Pennsylvania)	2		
St. Lawrence	15	0	
St. Louis	10	9	1
St. Martin's	0		
St. Mary-of-the-Woods (Indiana)	**		
St. Mary's (California)	2		
St. Mary's (Texas)	12		
St. Norbert	0		
St. Olaf	7		
St. Procopius	4		
St. Teresa (Minnesota)	2		
St. Thomas (Minnesota)	3		
Salem	3		
Sam Houston State Teachers College (Texas)	0		
San Diego State	19	0	
San Francisco	6		
San Jose State	0		
Scranton	19		
Seattle Pacific	4		
Seattle	0		
Seton Hill	0		
Shurtleff	1		
Simmons	2		
Skidmore	1		
Smith	3	3	
South Carolina	0		
South Dakota State	3		
South Dakota State School of Mines	7	*	
South Dakota	0	*	
University of South	3		
Southeast Missouri State	**		
Southeastern Louisiana	4		
Southeastern State (Oklahoma)	1		
Southern California	3	4	0
Southern Illinois	7		
Southern Methodist	5		
Southern Missionary	0		

	Bachelor's	Master's	Doctor's
Southwest Missouri State	2		
Southwest Texas State Teachers College	**		
Southwestern (Kansas)	4		
Southwestern Louisiana Institute	4		
Southwestern at Memphis	2		
Southwestern State (Oklahoma)	3		
Southwestern (Texas)	4		
Spring Hill	2		
Stanford	19	6	1
State College of Agriculture & Engineering (North Carolina)	*	2	
State A. & M. College of South Carolina	**		
Sterling	0		
Stetson	4		
Stevens Institute of Technology	2	6	
Suffolk	0		
Superior State Teachers College (Wisconsin)	2		
Susquehanna	2		
Swarthmore	9		
Sweet Briar	0		
Syracuse	18	10	0
Tampa	*		
Temple	9	1	
Tennessee Agricultural & Industrial State	**		
Tennessee Polytechnic Institute	12		
Tennessee	2	4	
Texas Christian	13	0	
Texas College of Arts & Industry	9	1	
Texas State College for Women	0		
Texas State (Houston)	0		
Texas Technological	4	1	
Texas	12	12	2
Texas Western	5		
Thiel	4		
Toledo	8		
Trinity (Connecticut)	4	*	
Trinity (Washington, D. C.)	2		
Trinity (Texas)	2		
Tufts	16	3	
Tulane	22	4	
Tulsa	10		
Tusculum	1		
Union (Nebraska)	6		
Union (New York)	11	3	
Union (Tennessee)	*		
Upper Montclair State Teachers College (New Jersey)	**		
Upsala	10		
Ursinus	2		
Utah State	7	0	
Utah	8	4	0
Valparaiso	4		
Vanderbilt	10	7	0
Vassar	1		
Vermont	5	0	
Villa Madonna	3		
Villanova	1	0	
Virginia Polytechnic Institute	19	1	
Virginia State College	4		
Virginia	4	5	6
Wabash	2		
Wake Forest	2		
Walla Walla	4		
Wartburg	4		
Washburn Municipal	2		

TABLE V.—(*Continued*)

	Bachelor's	Master's	Doctor's
Washington	2		
Washington & Jefferson	14	1	
Washington & Lee	2		
Washington Missionary	3		
State College of Washington	8	3	0
Washington (St. Louis, Missouri)	25	7	5
Washington (Seattle)	37	3	0
Wayne	9	2	
Waynesburg	6		
Wellesley	10	0	
Wells	0		
Wesleyan	2	3	
West Texas State Teachers College	0		
West Virginia Institute of Technology	0		
West Virginia	5	3	
Western College for Women (Ohio)	0		
Western Illinois State	8		
Western Kentucky State	6		
Western Maryland	5		
Western Michigan College of Education	5		
Western Reserve	4	0	
Western State College of Colorado	1		
Westmar	0		
Westminster (Missouri)	2		
Westminster (Pennsylvania)	4		
Wheaton (Illinois)	9		
Wheaton (Massachusetts)	**		
Whitewater State Teachers College (Wisconsin)	6		
Whitman	5		
Whittier	4		
Whitworth	3		
Wichita	5		
Willamette	4		
William Jewell	6		
William & Mary	8		
Williams	16	4	
Wilmington	4		
Wilson (Pennsylvania)	3		
Wilson Teachers (D. C.)	**		
Wisconsin	15	31	9
Wittenberg	0		
Wofford	6		
Woman's College of University of North Carolina	0		
Woodstock	4		
Wooster	5		
Worcester Polytechnic Institute	4		
Wyoming	7	4	
Xavier (Louisiana)	1		
Xavier (Ohio)	10		
Yale	18	16	14
Yeshiva	0		
Youngstown	3		

the periods September 1–August 31. The phenomenal increases ranging from 100 to 300 percent in a three-year period in the numbers of degrees in physics granted is the most striking feature of these data.

The increase in the number of institutions reported as granting the bachelor's degree is due only partly to the addition of departments which have established a physics curriculum. Most of the increase has been the result of ferreting out the institutions which actually offer an undergraduate physics major. Of especial interest is the tabulation in Table III of the doctorates in physics awarded in 1948–49.

The estimates for 1949–50 have been reduced from those given by the physics departments in the light of previous experience in these surveys. It has been observed that the estimates for bachelors are substantially correct but that those for masters are about 10 percent too high and those for doctorates have to be reduced to 60 percent of the indicated number of degrees.

The United States Office of Education has published[4] data for the numbers of degrees granted by all of the institutions of higher learning in the United States. It is interesting to compare the degrees awarded to physics students with those in other fields (Table IV). While the numbers of bachelor's and master's degrees in physics are exceeded in many other subjects the number of doctorates in physics places this science very high in the list.

In Fig. 1 the curve of doctoral degrees granted in the United States since 1912 strikingly shows the increased productivity in the training of professionally qualified physicists. The reductions because of World Wars I and II are clearly evident. It would seem that we have now reached a "normal" output of physicists at the doctorate level. The production of physics doctorates during the next few years will likely exceed that indicated by the curve of anticipated degrees to be granted.

The numbers of the various degrees granted by all of the physics departments of the United States which confer such degrees are shown in Table V. In this table of 500 institutions some 30 are listed which are believed to grant at least a bachelor's degree, although no data have been received from them, the numbers shown in the

[4] Circular 262, Federal Security Agency, U. S. Office of Education (1948–49).

TABLE VI. Institutions having the largest enrollments of undergraduate physics majors 1949–50.

California	411
(Berkeley 160)	
(Los Angeles 251)	
Massachusetts Institute of Technology	306
New York City College	275
Rensselaer Polytechnic Institute	205
Boston College	203
Washington (Seattle)	170
Colorado	147
Harvard	142
Siena College	137
Ohio State	133
Case Institute of Technology	130
Michigan	125
Purdue	125
The Pennsylvania State College	122
Illinois Institute of Technology	120
Maryland	115
Northeastern	114
Illinois	111
Fordham	106
Polytechnic Institute of Brooklyn	106

TABLE VII. Institutions having the largest numbers of graduate students in physics 1949–50.

California	365
(Berkeley 227)	
(Los Angeles 138)	
Maryland	284
New York University	262
(University Heights 30)	
(Washington Square 232)	
Columbia	253
Chicago	175
Cincinnati	175
Massachusetts Institute of Technology	172
Michigan	172
Ohio State	147
Illinois	146
Pennsylvania	133
Wisconsin	97
Polytechnic Institute of Brooklyn	95
Catholic University	92
Cornell	91
Harvard	90
Texas	90
Washington (Seattle)	90
Boston	88
Purdue	85

table being taken from the report of the United States Office of Education study. It is to be expected that some institutions offering a physics major may have been unintentionally omitted from this list. Corrections will gladly be made if omissions or errors are called to the attention of the author.

The "big 20" so far as numbers of undergraduate physics majors is concerned are listed in Table VI. A similar list of the institutions having the largest numbers of graduate students in physics is given in Table VII. It is frequently difficult for institutions to determine the proper number of graduate students to report, since many enrollments are for a single class by part-time people who are not expected to be candidates for a degree. In other departments nearly all of those enrolled are bona fide candidates for advanced degrees.

It is hoped that the publication of these data on physics enrollments and degrees conferred will be of value to those who are concerned with the professional problems of physicists, as well as to the advisors of prospective physicists in training. Competition for assistantships and fellowships for graduate study or even for admission to the leading graduate schools is becoming increasingly keen. While the number of such opportunities is large[5] there are not sufficient assistantships and fellowships for all applicants and the attention of students might well be directed to the less populous departments. The increased interest in physics has attracted to the undergraduate physics major and to graduate study many people who are unable or unwilling to do the grade of academic work necessary for success in this exacting field. But for those who reach the status represented by the usual physics doctorate there are currently adequate opportunities for satisfactory employment and professional development.

[5] T. H. Osgood, Am. J. Phys. **17**, 80 (1949).

On the Application of the Generalized Area-Moment Propositions

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The advantages in application of the generalized area-moment propositions developed by the author over the restricted forms of the same originally developed by Mohr are illustrated by detailed application to two standard cases in beam theory, in particular to the case of a simple beam with a concentrated load placed off center, and of an overhanging beam with a concentrated load placed at the free end.

IN a previous communication[1] the area-moment propositions of mechanics were put in more general forms than those originally given by Mohr and Greene. In particular, they were written in the forms

$$\theta_b=\theta_a+A/EI, \qquad \text{(Prop. I)}$$

$$y_b=y_a+(b-a)\theta_a+Ab^*/EI, \qquad \text{(Prop. II)}$$

where θ_a represents the slope of the elastic curve at the point corresponding to $x=a$; θ_b represents similarly the slope at $x=b$; y_a represents the deflection of the elastic curve at $x=a$; y_b similarly the deflection at $x=b$; A represents the area under the moment curve between the limits a and b; b^* represents the distance of the centroid of the same area from the line $x=b$ (b^* is *positive* if the centroid lies to the *left* of the line $x=b$, and negative if to the right); and, finally, the product EI represents, as usual, the flexural rigidity of the beam considered.

Notably, in the more general form, the second area-moment proposition also involves, in addition to the deflection, the slope of the elastic curve, *hence can be used to determine slopes when deflections are known*. This fact permits the simple solution of the (otherwise somewhat troublesome) standard case of a simply supported beam with a concentrated load placed off center, and that of an overhanging beam, as well as many others. We shall illustrate the method by applying it in detail to the two cases cited.

Simple Beam with Concentrated Load Off Center

Let a concentrated load P be applied transversely to a simply supported beam at a distance c from the left-hand support. Denote the distance of the load from the right-hand support by d and the length of the beam by l. The load and moment diagrams will then be those of Fig. 1.

In order to determine the slope θ_0 of the elastic curve at the *left*-hand support, we apply directly (Prop. II) setting $a=0$ and $b=l$, whence we have

$$0=0+l\theta_0+c^2dP(d+c/3)/2lEI + cd^2P(2d/3)/2lEI,$$

which, on simplifying becomes

$$\theta_0=-Pcd(c^2+3cd+2d^2)/6l^2EI.$$

By substituting for c its equivalent $l-d$, the last equation reduces readily to the form in which it is usually given, namely,

$$\theta_0=-Pd(l^2-d^2)/6lEI.$$

In order to determine the slope at the *right*-hand support, we can now make use of (Prop. I) setting $a=0$ and $b=l$, and have further

$$\theta_l=\theta_0+Pcd/2EI,$$

which, on substituting for θ_0 the value previously found and simplifying, yields

$$\theta_l=Pcd(2c^2+3cd+d^2)/6l^2EI.$$

By substituting for c its equivalent $l-d$, the last equation can be reduced readily to the form in which it is usually given, namely,

$$\theta_l=Pcd(2l-d)/6lEI.$$

(The latter slope could have been found independently by applying Proposition II directly, setting for this purpose $a=l$ and $b=0$.)

In order to determine the deflection y_c under the load P, we apply Proposition II, setting $a=0$ and $b=c$, whence we have

$$y_c=0+c\theta_0+Pc^3d/6lEI.$$

[1] A. W. Simon, Am. J. Phys. 16, 409–410 (1948).

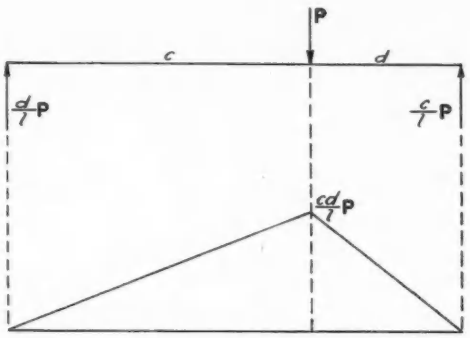

FIG. 1. Load (upper) and moment (lower) diagram of simply supported beam with concentrated load between supports.

from which, after substituting for θ_0 the value already found, we obtain

$$y_c = -Pc^2d^2/3lEI.$$

Overhanging Beam

Let a beam be supported at the left end and also at a distance c from the left end, while a single concentrated load P is placed at the extreme right end of the beam, which overhangs the corresponding support a distance d. The load and moment diagrams will then be those of Fig. 2.

In order to find the slope θ_0 at the *left* support, we apply directly Proposition II, setting $a=0$ and $b=c$, whence we obtain

$$0 = 0 + c\theta_0 - Pc^2d/6EI,$$

or, solving for θ_0,

$$\theta_0 = Pcd/6EI.$$

In order to find the slope θ_c at the *right*-hand support, we now apply Proposition I, setting $a=0$ and $b=c$. This yields

$$\theta_c = \theta_0 - Pcd/2EI,$$

which, on substituting for θ_0 the value previously found, becomes

$$\theta_c = -Pcd/3EI.$$

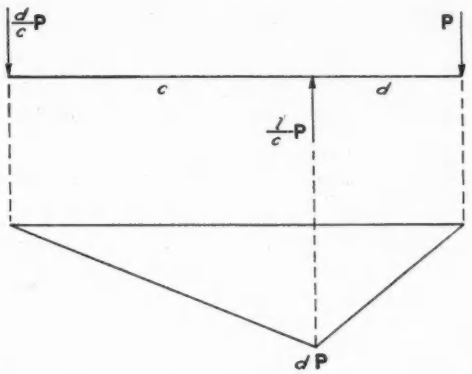

FIG. 2. Load (upper) and moment (lower) diagram of overhanging beam with concentrated load at the free end.

To find the slope θ_l at the extreme right end, we apply Proposition I again, this time setting $a=c$ and $b=l$, whence we obtain

$$\theta_l = \theta_c - Pd^2/2EI,$$

from which, on substituting for θ_c the value already found, we obtain

$$\theta_l = -Pd(2l+d)/6EI.$$

In order to calculate the deflection under the load, we now apply Proposition II, setting $a=c$ and $b=l$, and have

$$y_l = 0 + d\theta_c - Pd^3/3EI,$$

from which, on substituting for θ_c the value previously found and simplifying, we obtain

$$y_l = -Pld^2/3EI.$$

From the examples given it is readily seen that the generalized area-moment propositions afford a much simpler and more direct method for solving beam problems than the restricted forms of the same usually given in textbooks on strength of materials. As is well known, in application the restricted forms alone are usually not sufficient to solve the problem but additional geometrical constructions and mathematical artifices are required.

Every cause produces more than one effect.—HERBERT SPENCER.

The Capture of Orbital Electrons by Nuclei (*K*-Capture)

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(Received June 26, 1950)

The paper presents a review of recent work on *K*-capture of orbital electrons by nuclei, including a simple outline of the underlying theory and a discussion of the conditions under which the phenomenon occurs. A brief description is given of the instruments and methods used to observe *K*-capture.

I. The Positive Electron (Positron)

ALL the electrons emitted by the naturally occurring radioactive nuclei are negatively charged, as are those electrons associated with extranuclear processes. However, since the discovery in 1934 of artificially produced radioactivity,[1] many isotopes have been observed to decay with the emission of a positively charged particle whose ratio of charge to mass (e/m) is the same as that of the electron.

This particle does not continue to exist in nature but is destroyed by an interaction with a negative electron. Most often this process of destruction is found to occur after the positively charged particle has come to rest. Therefore conservation of both energy and momentum requires two gamma-rays to be emitted in exactly opposite directions, each having an energy equal to the rest energy of a single electron ($h\nu = m_0c^2$). Because this radiation is associated with the annihilation of a positron (positively charged electron) and a negatron (negatively charged electron), it has been appropriately named annihilation radiation.

An adequate explanation of the positron and the phenomena associated with it seems to exist in Dirac's relativistic theory of the electron.[2] This is sometimes referred to as Dirac's "hole" theory of the positron. The results of this theory indicate that the energies which a free negatively charged electron may have must be given by the equation $E = \pm[(m_0c^2)^2 + p^2c^2]^{\frac{1}{2}}$, where p is the momentum of the electron. This says that such a free electron must have an energy greater than $+m_0c^2$ or less than $-m_0c^2$, these two values being obtained by setting the momentum p of the electron equal to zero. Classically the existence of an electron in a negative energy state ($E < -m_0c^2$) has no physical meaning. However, quantum-mechanically it can be shown[2] that an external field can cause transitions from a state of positive energy to a state of negative energy, such that, provided Dirac's theory is not incorrect, the negative energy states must assume some physical meaning. To do this Dirac assumes that under normal conditions every negative energy state is occupied by a single electron. Thus, according to this hypothesis there exists everywhere a continuous sea of negatively charged electrons in negative energy states. Since physical measurements record only changes in the physical picture, the fact that all possible negative energy states are filled precludes the possibility of observation. However, should an electron be removed from one of these negative energy states the "hole" which remains will disrupt the *status quo* and will appear to be a positively charged particle. It will appear to have positive energy since to make it disappear requires that there be added to it an electron with negative energy. Since the largest energy which this hole can have is $E = -m_0c^2$, the observed positron appears to have a rest mass just equal to that of the negatron.

Using the Dirac theory as a basis, one may set up the process of positron decay of a radioactive nucleus as a case in which a proton in the nucleus captures one of the negative energy state electrons, thereby transforming itself into a neutron. The positively charged "hole" thus produced in space carries with it a certain amount of kinetic energy and appears to be a particle moving through space. In order to conserve energy and

* Assisted by the joint program of the ONR and AEC.
[1] I. Curie and F. Joliot, Compt. rend. **198**, 254 (1934).
[2] P. A. M. Dirac, *The Principles of Quantum Mechanics*, (Oxford University Press, London, 1947), third edition, pp. 272 ff. W. Heitler, *The Quantum Theory of Radiation*, (Oxford University Press, London, 1944), second edition, pp. 186 ff.

spin, a neutrino is considered to be produced in the process, this neutrino carrying with it the energy of the transition not taken by the positron.[3]

II. Orbital Electron Capture by the Nucleus

One can argue that the negatron required to transform a proton into a neutron in the beta-decay process need not necessarily come from a negative energy state. The proton might conceivably capture a negatron from a positive energy state since a large number of such particles do exist throughout space.[4]

In the case of positron decay the positively charged "hole" in space provides a measurable radiation. However, in the process of negatron capture from a positive energy state, no measurable nuclear radiation is evolved since the neutrino is the only emitted particle. This difficulty can be, however, at least partially resolved.

In any case, the basic criterion for capture by the nucleus is that the negatron be in the immediate vicinity of the nucleus. This presents no problem in the "hole" theory of positron decay since, according to it, negatrons in negative energy states exist throughout space. However, negatrons in positive energy states are not so generally distributed throughout space. Furthermore, they are strongly affected by the Coulomb force exerted upon them by the positively charged nuclei.

Because of these facts the only group of electrons which regularly come close enough to the atomic nucleus to be captured, should such capture be energetically possible, are those electrons, bound to the atom, whose orbital angular momentum is zero. Within an atom the electrons are distributed within a system of shells, the number of such shells in a neutral atom depending upon the number of electrons within that atom.[5] Each of the atomic shells contains two electrons having zero orbital angular momentum. However the electrons in the innermost shell (K-shell according to x-ray notation) spend a larger percentage of their time in the immediate vicinity of the nucleus than do the electrons from the outer shells (L, M, N, etc., shells according to x-ray notation). Because of this fact there is a larger probability for capture of a K-shell electron by the nucleus than there is for capture of an electron from an outer atomic shell. Likewise there is a greater probability of capture from the L shell than from the M shell, etc. In fact, the probability for capture decreases quite rapidly as one progresses toward the outer shells of the atom. The larger probability for capture of course can be translated into the statement that, within any given group of radioactive nuclei capable of decay by orbital electron capture, a predominate number of the captures will come from the atomic K-shell. Because of this the process is quite commonly referred to as K-capture.

The fact that a large percentage of the orbital captures are from the K-shell is perhaps rather fortunate for, even though no measurable nuclear radiation evolves from the process, the subsequent filling of the empty space in the K-shell produces a measurable radiation, either an x-ray or an Auger electron.[6] This radiation will be characteristic of the product (daughter) atom since, prior to the emission of this radiation, the capture of an electron by the nucleus will have transformed a proton into a neutron within the nucleus and will have changed the atomic charge from Z to $Z-1$.

Using the experimental evidence available in 1938, Alvarez[7] concluded that the large amount of characteristic x-radiation, detected in the presence of certain radioactive transitions, is almost certainly positive proof for the existence of the K-capture process. In fact, in certain

[3] For an elementary discussion of the beta-decay hypothesis see C. S. Cook and G. E. Owen, Am. J. Phys. **18**, 453 (1950), and for a more thorough study of the process see E. J. Konopinski, Revs. Modern Phys. **15**, 209 (1943).

[4] That such a process should occur was first postulated by H. Yukawa and S. Sakata, Proc. Phys.-Math. Soc. Japan **17**, 467 (1935); **18**, 128 (1936).

[5] For a study of the distribution of electrons within an atom, see G. Herzberg, *Atomic Spectra and Atomic Structure* (Dover Publications, New York, 1944), pp. 40–45, or L. C. Pauling and E. B. Wilson, *Introduction to Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1935), pp. 139–150.

[6] The name arises from the name of the discoverer of the process, P. V. Auger. Since such electrons can be emitted by an atom, the probability for the release of a characteristic x-ray when an electron falls into the vacated K-shell is less than unity. This probability for the release of an x-ray is called the fluorescence yield of the atom. For further details on this subject the reader is referred to A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand Company, Inc., New York, 1935), pp. 477–492.

[7] L. W. Alvarez, Phys. Rev. **54**, 486 (1938).

cases, such as the long-lived titanium activity mentioned by Alvarez there is no detectable emission except characteristic x-rays and Auger electrons.

III. Energy Requirements for Particle Emission From the Nucleus and for Orbital Electron Capture by the Nucleus

The fact that positron emission may be completely absent, even though orbital electron capture is present, may be shown purely by reason of available energies. The emission of a positron must carry away from the nucleus a minimum energy equal to that of the rest energy of the positron (m_0c^2).

However, for the purpose of drawing nuclear energy level diagrams, the atom as a whole, rather than the bare nucleus, is usually considered.[8]

In order to keep the total atomic charge neutral following the emission of a positron from the nucleus, a negative electron must be lost by one of the outer shells of the atom. The atom as a whole, therefore, loses a *minimum* energy equal to twice the rest energy of the electron.

On the other hand, in the case of K-capture the nucleus loses a single positive charge merely by taking a negative electron from its own K-shell. Then the only energy which must be lost by the atom is that given up in the form of x-rays and Auger electrons when the extranuclear structure rearranges itself into the form of an atom of one lower atomic number. If, however, capture is made from some shell other than the K-shell (see Sect. IV) less energy will be available for release during the rearrangement of the atomic electrons. Although the frequency of occurrence is small, it is distinctly possible for capture to take place from a shell having small binding energy. In the limit, therefore, the plausibility of nuclear electron capture without the release of an appreciable amount of energy must be considered.

The conditions prescribed in the last two paragraphs are illustrated in Fig. 1. In order to be the product of a radioactive decay the ground-state energy level of the atom whose nuclear charge is $Z-1$ must exist in the regions covered by the diagonal lines. Should its ground state lie higher than the zero energy level, orbital electron capture is no longer energetically possible.

Perhaps the simplest case to illustrate this situation in which orbital electron capture becomes impossible purely because of the absence of sufficient energy is that of the hydrogen atom. The question often arises, especially in connection with an undergraduate class in modern physics, as to why the electron, being negatively charged, isn't captured by the positively charged proton at the center of the hydrogen atom. Since the rest energy (Mc^2) of any particle is proportional to its mass, a measurement of the relative masses of a number of particles may also be considered as a measure of their respective rest energies. The capture of an orbital electron by the proton at the center of the hydrogen atom requires the formation of a neutral nuclear particle, namely a neutron (and of course, to conserve spin a neutrino must also be formed).[3]

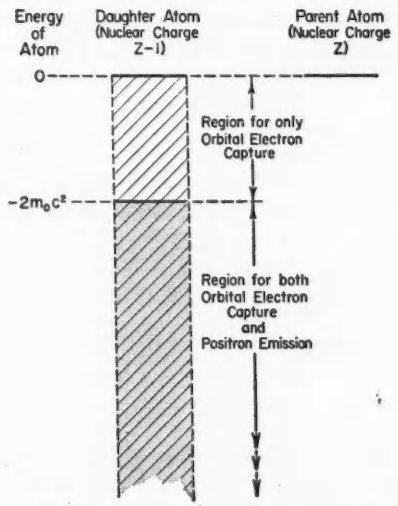
Energy of Atom
Daughter Atom (Nuclear Charge Z−1)
Parent Atom (Nuclear Charge Z)
0
−2m₀c²
Region for only Orbital Electron Capture
Region for both Orbital Electron Capture and Positron Emission

FIG. 1. Diagrammatical representation of energy conditions necessary within the atom for orbital electron capture and for positron emission. The ground-state energy level of the parent atom is represented by the solid horizontal line on the right. If the ground-state energy level of the daughter atom is located in the lined region between 0 and $-2m_0c^2$ energy, there is insufficient energy available for positron emission (as explained in text) and the parent atom may decay only by orbital electron capture. However, if the ground-state energy level of the daughter atom falls in the shaded region below $-2m_0c^2$, the parent atom may decay either by orbital electron capture or by positron emission.

[8] A. L. Hughes, Am. J. Phys. 16, 415 (1948).

However, the mass of a neutron is greater than the combined mass of a proton and an electron making orbital capture of an electron by a hydrogen nucleus energetically impossible. In fact, calculations show that the decay of a neutron into a proton, a negatron, and an antineutrino should be of the allowed type with an expected half-life of less than 30 minutes. Recent experiments[9] indicate that the neutron decays by such emission with a half-life somewhere in the range from 9 to 25 minutes. However, if the negative electron has sufficient kinetic energy the transformation of a proton to a neutron can be achieved. A relatively large kinetic energy can be acquired by the negatron by either electrostatic (Van de Graaff and other allied instruments) or electromagnetic (betatron and synchrotron) means. If this added kinetic energy is sufficient to give the electron-proton system a total energy equal to or greater than the rest energy of the neutron, then there is a certain probability that the proton-electron system will transform into a neutron and neutrino. However, the fast moving electron spends so little time in the immediate vicinity of the proton that, even though energetically possible, the probability of their combination is extremely small.

Another similar, but what might be called reverse, process is one in which a neutrino is captured by a proton with the resultant production of a neutron and positive beta-particle. Here again the probabilities are extremely small. Neither process has been observed experimentally although recent experiments[10] have become sufficiently sensitive to indicate that the probability of the former process occurring cannot be much larger than is theoretically predicted.

IV. Experimental Verification of Orbital Capture From Shells Other Than the *K*-Shell

As has been indicated above, orbital capture of electrons having zero orbital angular momentum is theoretically possible from any of the atomic shells. One expects the capture probability to be much larger for *K*-shell electrons than for *L*-shell electrons. This is a consequence of the large density of *K*-shell electrons in the immediate vicinity of the nucleus; whereas the density of *L*-shell electrons, and of electrons in further shells is smaller near the nucleus. In order to determine experimentally the respective capture probabilities for each of the shells certain special conditions, which are rather difficult to obtain, are required. The most ideal conditions for such an experiment include the criterion that the isotope in question decay only by the *K*-capture process. The simplest such isotope is Be^7. Metallic Be has four atomic electrons, two in the *K*-shell and two in the *L*-shell. Certain molecular structures involving beryllium, however, effectively rob, at least for part of the time, the beryllium atom of an *L*-shell electron and attach it to some other atom in the molecule. The more electronegative the atom (or atoms) that is compounded with the beryllium to form a molecule, the greater the percentage of time that the beryllium atom is stripped of its *L*-shell electron.

One measures the relative amounts of *K*- and *L*-capture through studies of the respective half-lives for the two cases. The beryllium metal, having a greater density of electrons in the immediate vicinity of the nucleus, will have a larger probability for electron capture and therefore will have a shorter half-life. Two independent studies[11,12] of Be^7 using beryllium metal as the non-ionized Be^7 source and BeF_2 as the source of ionized Be^7 have been made. Calculations indicate that the partial ionization of the beryllium in the BeF_2 molecule is quite large, being about 87 percent. The results obtained by both groups are qualitatively the same, the BeF_2 having the longer half-life, indicating the presence of electron capture from the atom's *L*-shell as well as from its *K*-shell. However, the numerical results are about five times larger for the group in France $[(T_{BeF_2}-T_{Be})/T_{Be}=0.01 \pm 0.003]$ than for the group in California $[(T_{BeF_2}-T_{Be})/T_{Be}=0.00168\pm 0.00019]$. The letter T represents the experimentally determined half-life for the Be metal or BeF_2 molecule in accordance with the subscript used. These results

[9] J. M. Robson, Phys. Rev. **77**, 747 (1950); **78**, 311 (1950); Snell, Pleasonton, and McCord, Phys. Rev. **78**, 310 (1950).

[10] D. Saxon, Phys. Rev. **76**, 986 (1949).

[11] Leininger, Segrè, and Wiegard, Phys. Rev. **76**, 897 (1949).

[12] Benoist, Bouchez, Daudel, Daudel, and Rogozinski, Phys. Rev. **76**, 1000 (1949); Bouchez, Daudel, Daudel, and Muxart, Compt. rend. **227**, 525 (1948).

show that the amount of capture from shells other than the K-shell is extremely small, thus making the name K-capture, as now applied to the process, quite valid.

V. Experimental Methods for Showing the Presence of Orbital Electron Capture

The fact that the orbital electron capture process provides no measurable nuclear radiations, but only the extranuclear x-rays and Auger electrons, makes the detection of this process somewhat different and more difficult than the detection of the other types of radioactive decay. Any direct observation must provide a means for measuring either the x-rays or Auger electrons. The low energy of these radiations makes them readily absorbable by any material which is interposed between source and detector. However, a number of methods for detecting these two types of radiations have been used and will be discussed in the following paragraphs.

The existence of Auger electrons may be shown by means of a beta-ray spectrometer providing the window of the Geiger-Mueller counter used as the detecting device in the spectrometer is sufficiently thin to pass these electrons. For most of this work a laboratory prepared window made from zapon, Nylon, Formvar or similar material is required. The thinnest mica window counters are usually so thick that the Auger electrons cannot penetrate them.

The Auger electrons will appear in the beta-ray spectrum as a low energy, almost monoenergetic peak. The energy of the ejected electron will be equal to that of the atomic transition producing the radiation less the binding energy of the electron prior to its ejection from the atom. Thus an electron from the L-shell which gathers its energy from a transition of another electron between the L and K-shells will leave the atom with a kinetic energy $E_K - 2E_L$, where E_K and E_L represent the binding energies of electrons in the K and L-shells, respectively, of the atom. The corresponding x-ray would have an energy $E_K - E_L$.

The momentum spectrum of the negative electrons emitted by Cu^{64}, as obtained[13] in a beta-ray spectrometer using a Geiger-Mueller counter as the detecting device, is shown in Fig. 2. The G-M counter window would not pass electrons whose energy was less than 3 kev. The low energy (about 7 kev) Auger electron line and the negative beta-particle spectra are distinctly visible. Generally no negative beta-particle spectrum is present. However, Cu^{64} is one of those radioactive elements which decays through emission of either a positive or a negative beta-particle. This spectrum will be discussed further in connection with the determination of the ratio of the abundance of K-capture to the abundance of positron emission in Sect. VI.

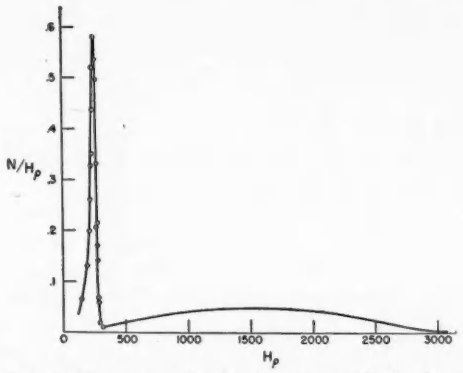

FIG. 2. Momentum distribution of negatively charged electrons emitted by radioactive Cu^{64} as recorded by Cook and Langer (see reference 13). The low energy peak is a measure of the Auger electrons associated with K-capture by this nucleus. The broader distribution is the spectrum of the negative beta-particles emitted by the Cu^{64} nucleus. This gives some idea of the high intensity of the Auger electron radiation associated with the K-capture process.

The existence of monoenergetic groups of x-rays, if characteristic of the appropriate element, may also provide strong evidence that the process of orbital electron capture occurs. Again, just as in the case of the Auger electrons, experimental verification of the existence of monoenergetic x-ray groups presents somewhat a problem. However, two types of experiments have been used with considerable degree of success.

The first of these methods makes use of absorption techniques. It is the process known by the name of critical absorption.[14] Although the

[13] C. S. Cook and L. M. Langer, Phys. Rev. **74**, 1241 (1948).

[14] For a more detailed discussion of the method of critical absorption as applied to x-rays, see any atomic or modern physics book. For example the reader is referred to

absorption of x-rays by any material is relatively high because of the low energy of the radiation quanta, there are distinct breaks in the value of the coefficient of absorption for any element. These breaks occur at energies corresponding to the binding energies of the atomic electrons in their respective shells.

If a radioactive source emits monoenergetic groups of x-rays, as in K-capture, then one should be able to detect this fact by using successive sets of absorbers made from an appropriate series of elements. Generally elements of higher atomic number will absorb the radiation somewhat more strongly than those of lower atomic number. However, when an electron shell exists in two adjacent elements such that its binding energy in one is less than the x-ray energy and in the other is greater than the x-ray energy, a distinct break occurs in the value of the absorption coefficient for that x-ray energy.

Even though this method has been used successfully[7] to show the existence of the necessary characteristic x-rays, it is somewhat inadequate from the point of view of being a simple experiment to perform. It is ofttimes exceedingly difficult to obtain similar solutions or foils for enough adjacent elements of the periodic table to carry out a completely successful experiment. This inability may be caused either by the different physical and chemical characteristics of adjacent elements or by the fact that some desired element may be so rare that the experiment becomes impractical from a strictly financial point of view.

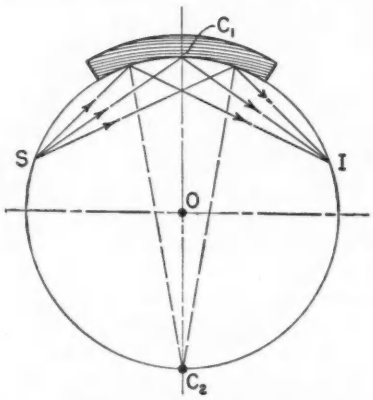

FIG. 3. The reflection type curved crystal spectrometer. For proper operation the surface of the crystal is ground into the arc of a circle (focal circle) whose center is O after the crystal has been bent such that its atomic reflecting planes are concentric about the center C_2 on the circumference of the focal circle. The radiations from a line source S on the focal circle will be focused by the crystal into a line image I on the same circle.

A now widely adopted method for localizing the element of origin of the x-radiation following the K-capture process is that of determining the diffraction pattern formed by a crystal spectrometer. Here a precise measurement of the wavelength of the x-rays is obtained. The chief problem which arises in this type of experiment is the problem of intensity. In fact this problem is so acute in the study of x-radiation from radioactive substances that none of the earlier types of crystal spectrometers can produce satisfactory results.

However, the problem of intensity has been overcome with a fair degree of success through use of a curved crystal in the spectrometer.[15] The curved crystal provides a focusing property which brings into the detector monochromatic radiation which has been emitted from the source within a relatively large solid angle. The crystal may be cut and bent in either of two ways. One way is for use in a reflection type spectrometer, the other for use in a transmission type spectrometer.

The reflection type instrument is shown in Fig. 3 and its method of operation discussed in the caption for that figure. This type of instrument has been used successfully by Abelson[16] for the study of characteristic x-radiation associated with the K-capture process.

The reflection type instrument has not been used extensively, however, largely because of one advantage which the transmission type curved crystal holds over it. Whereas the reflective use of the crystal requires both source and detector to be confined to regions whose physical dimensions are small, the transmissive use of the crystal allows one of these to be spread

G. E. M. Jauncey, *Modern Physics*, Third Ed. (D. Van Nostrand Company, Inc., New York, 1948), pp. 316–321; J. D. Stranathan, *The Particles of Modern Physics* (The Blakiston Company, Philadelphia, 1942), pp. 286–290; F. K. Richtmyer and E. H. Kennard, *Introduction to Modern Physics*, Fourth Ed. (McGraw-Hill Book Company, Inc., New York, 1947), pp. 501–506.

[15] J. W. M. DuMond, Rev. Sci. Instr. **18**, 626 (1947); earlier references are given in this work.

[16] P. Abelson, Phys. Rev. **56**, 753 (1939).

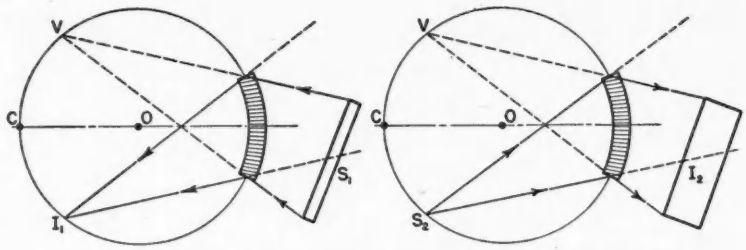

FIG. 4. The transmission type curved crystal spectrometer. In this case the atomic reflecting planes converge radially to a point C on the focal circle. The surface is again, as in the reflection type, cut such as to form a circle having its center at O. Under these conditions the radiations from an extended source S_1 may be focussed into the line image I_1, or a line source S_2 may be recorded by an extended detecting device I_2.

over a relatively extended region of space. Figure 4 and its caption describes this use of this curved crystal. The extended source type has been used at Ohio State[17] and Iowa State[18] while the extended detector type has been developed and used extensively by DuMond and his co-workers[19] at the California Institute of Technology. This instrument has been improved to the extent that it is now possible to measure precisely the wavelengths of gamma-rays up to an energy of 1.3 Mev.[19,20]

VI. Measurements of the Relative Amounts of Orbital Capture Occurring in a Radioactive Isotope

We have seen that it is possible for certain radioactive nuclei to decay either by the emission of a positron or by the capture of an orbital electron. The theory of beta-decay has now been developed in such detail that, for a given transition, predictions may be made of the relative number of nuclei which capture an orbital electron to those emitting positron radiation.[21] Such values are customarily called K-capture/positron ratios. One of the chief problems which still remains in study of orbital electron capture is the experimental verification that these predicted K-capture/positron ratios agree with the true ratios. In recent years a number of such studies have been made.

The most direct attack upon the problem involves a comparison of the number of characteristic x-rays (or Auger electrons) with the number of positrons emitted by a source composed of a single type of radioactive nucleus.

Attempts to compare the characteristic x-ray intensity with the positron intensity in order to determine precise K-capture/positron ratios recently have been made by two groups, one in France,[22] the other in Switzerland.[23] Both groups have used the decay of Cu^{61} into Ni^{61} and of Cu^{64} into Ni^{64} as the transitions upon which to perform their experiments.

The chief difficulty which arises in an experiment of this type is the large number of corrections necessary in order to obtain an absolute orbital electron capture rate. The French group itemizes the corrections in the x-ray data so as to account for losses of intensity caused by absorption within the source, by absorption in the air between the source and the Geiger-Mueller counter, absorption by the counter window, absorption by the gas (usually argon) within the counter, and by the fact that the efficiency of the counter is less than 100 percent (caused both by an intrinsically low efficiency for counting quanta of electromagnetic radiation and by the fact

[17] Edwards, Pool, and Blake, Phys. Rev. **67**, 150 (1945).
[18] R. H. Stokes, AECD-1843 (1948).
[19] J. W. M. DuMond, Phys. Rev. **79**, 210 (1950).
[20] Lind, Brown, and DuMond, Phys. Rev. **76**, 1838 (1950). This paper gives references to previous work.
[21] A relatively detailed treatment of the theory from which K-capture/positron branching ratios may be calculated is given by Bouchez, de Groot, Nataf, and Tolhoek, J. phys. et radium **11**, 105 (1950). Calculations of these ratios for certain nuclei have been made and the results prepared in the form of graphs by Feenberg and Trigg, Revs. Modern Phys. **22**, 405 (1950). The basic theory upon which these calculations are made is also discussed by Feenberg and Trigg. Both articles give references to earlier work on the subject.
[22] R. Bouchez and G. Kayas, J. phys. et radium **10**, 110 (1949); Bouchez, de Groot, Nataf, and Tolhoek, Physica **15**, 863 (1949), J. phys. et radium **11**, 105 (1950).
[23] Huber, Ruetsche, and Scherrer, Helv. Phys. Acta **22**, 375 (1949).

TABLE I. Comparison of the experimentally determined and theoretically predicted K-capture/positron ratios for a number of radioactive isotopes.

Isotope	Observed K/β^+	Theoretically predicted value[b]
Na^{22}	0[a, b]	0.10
V^{48}	0.72[a]	0.65
Mn^{52}	1.85[a]	1.78
Co^{58}	5.89[a]	5.01
Cu^{61}	0.55[c]	0.29–0.52[i]
Cu^{61}	0.38–0.43[d]	0.29–0.52[i]
Cu^{61}	0.32[e]	0.29–0.52[i]
Cu^{64}	2.65[c]	2.08
Cu^{64}	1.75[g]	2.08
Cu^{64}	2.32[f]	2.08
Cd^{107}	320.[g]	342.

[a] See reference 30.
[b] H. Maier-Leibnitz, Z. Physik **122**, 233 (1944).
[c] See reference 22, R. Bouchez and G. Kayas.
[d] See reference 22, Bouchez, de Groot, Nateť, and Tolhock; R. Bouchez, private communication.
[e] See reference 23.
[f] See reference 34.
[g] See reference 31.
[h] Calculations of theoretical values are based upon the calculations of G. Trigg and E. Feenberg; Phys. Rev. **77**, 756 (1950); Revs. Modern Phys. (see reference 21).
[i] The lower value is the value quoted until recently as determined under the assumption that Cu^{61} follows a simple disintegration scheme to the ground state of Ni^{61}; the complex disintegration scheme which now appears to exist for Cu^{61} [see Owen, Cook, and Owen, Phys. Rev. **78**, 686 (1950)] leads to the higher value.

that a solid angle less than 4π-steradians is subtended by the counter as viewed from the source). In addition, since only the x-ray quanta are detected a correction for the fluorescence yield[6] must be made.

Determining an absolute counting rate for the positrons is somewhat easier since a number of methods exists. First of all the use of a G-M counter in the same fashion as for the x-rays does not require quite so many large corrections. Secondly, the use of an ionization chamber for comparison of the intensity of the positron radiation with the beta-radiation from a standard source is possible, and was used by the French group with RaE being the standard source. Finally the annihilation radiation can also provide a measure of the intensity of the positron radiation. It was this annihilation radiation which was measured by the group from Zurich.

The results of these various measurements are given in Table I. When one stops to consider the necessity of so many required corrections, there is really nothing remarkable about the fact that the values obtained by this method should vary so widely. With so many corrections it seems quite possible that there could be several small errors or even a single major error. However, it should probably be indicated that the most recent value determined[22] by this method is in very good agreement with the value determined by Reynolds by an entirely different method. This other method will be discussed in a later paragraph.

The use of the Auger electrons, rather than the x-rays, as a measure of the number of orbital electron transitions provides a somewhat more direct measurement but still presents its own difficulties. Theoretically it should be possible to obtain a spectrum in a beta-ray spectrometer showing both the Auger electrons and the positrons. Determination of the relative areas under the curves obtained in these two spectral measurements then provides a measure of the relative intensities of the two types of radiations. If all the particles were able to get into the detecting device used in the spectrometer, then the only corrections necessary would be those required by the fluorescence yield factor and the efficiency of the detecting device. Of those detectors used in beta-ray spectrometers the one whose efficiency is most nearly constant throughout the entire range of electron energies is the G-M counter. For it, however, a correction must be made at Auger electron energies for the relative transmission of the window. The thinnest windows used on the normal beta-spectrometers usually stop all electrons whose energies are less than 4 or 5 kev. However, this does not mean that they allow all electrons of energy greater than 4 or 5 kev to pass. On the contrary a window of this type allows only a certain fraction of the electrons impinging upon it to pass through, this fraction growing larger as the energy of the electron grows larger.[24–26] An accurate window thickness correction curve would lead to more accurate results.

The results of an attempt[13] at such a K-capture/positron ratio determination is shown in Fig. 2. The almost monoenergetic electron peak in the low energy region is caused by the Auger electrons associated with the K-capture process in Cu^{64}. The continuous spectrum is the negatron

[24] M. Ference and R. J. Stephenson, Rev. Sci. Instr. **9**, 246 (1938).
[25] Struken, Heller and Weber, Phys. Rev. **78**, 327 (1950).
[26] C. S. Cook and C. H. Chang, Phys. Rev. **79**, 244 (1950).

spectrum of this isotope. Since the area under each curve gives a relative value for the intensity of each type of radiation, the ratio of the areas should give the ratio of Auger electron to negatron intensity. The ratio of negatron to positron emission in Cu^{64} is known[27] to be 2, so the ratio of Auger electron to positron intensity can be determined. If the negatron spectrum were not present, as is the case normally, a measure of the ratio of the areas of the Auger line (after necessary corrections) and of the positron spectrum (using the same source) would provide sufficient data for the calculation.

It is possible that much of the need for a window transmission correction can be eliminated through use of a windowless counter spectrometer of the type which has been developed by Langer and his co-workers at Indiana.[28]

It may also be possible to overcome some of these difficulties through use of a proportional counter spectrometer.[29] Here one introduces the radioactive substance into a large proportional counter. The radiated particles produce ionization in proportion to their energy and the amount of ionization is measured by the size of the electrical pulse produced in the counter. Thus one can get a spectrum similar to that obtained in the magnetic analysis type of beta-spectrometer.

This type of determination of K/β^+ should need only a single correction, that of fluorescence yield. Basically, the only other difficult task is putting the radioactive source into a form such that it can be used in the proportional counter. No successful measurements of K/β^+ using this technique have yet been completed.

Because of the impracticability of quantitative measurement of the x-rays or Auger electrons, Good, Peaslee, and Deutsch[30] and a group at Zurich[31] used indirect methods for obtaining K-capture/positron ratios. The special nature of these indirect ratio determinations limits the number of isotopes which can be so measured.

The Zurich group found that Cd^{107} (6.7-hr half-life) decays into a metastable state of Ag^{107} (44.3-sec half-life). The transition from the metastable state to the ground state of Ag^{107} releases its energy almost exclusively in the form of internal conversion electrons. From a measurement of the number of internal conversion electrons, the total number of radioactive transitions, including both those decaying by positron emission and those decaying by K-capture, can be obtained. A direct measure of the number of positrons then provides a value for $(K+\beta^+)/\beta^+$, where K is the number of K-capture transitions and β^+ the number of positron transitions.

Although an absolute measurement of the number of internal conversion electrons and positrons is extremely difficult, the ratio of these two quantities may be calculated in a rather simple manner by comparing the areas under the internal conversion line and under the positron momentum spectrum as determined in a beta-ray spectrometer.

This is the same method of comparison as suggested for a direct observation of the Auger electron/positron ratio. However, the energy of the internal conversion electrons is generally (and is in this case) greater than that of the Auger electrons. Since the transmission of the G-M counter window will now be almost 100 percent, negligible corrections are required.

These measurements resulted in a branching ratio $K_{Cd^{107}}/\beta^+_{Cd^{107}}=\lambda_{Cd^{107}}=320\pm20$. This agrees quite closely with theory which predicts 342 for this value.

The MIT group[30] used a somewhat similar technique measuring, however, gamma-ray intensities instead of internal conversion electron intensities. This method is limited to K-capture and positron transitions followed by gamma-radiation. For this experiment a source is enclosed in a tiny brass cylinder sufficiently thick to stop all positrons. The only measure of the positron intensity may be made through detection of the annihilation radiation. The source and one G-M counter are fixed in position, and a second G-M counter is set up at a fixed distance on a rod which rotates about the source as an axis (as shown in Fig. 5). Either counter used

[27] C. S. Cook and L. M. Langer, Phys. Rev. **73**, 601 (1948).

[28] Langer, Motz, and Price, Phys. Rev. **77**, 798 (1950).

[29] Curran, Angus and Cockroft, Phil. Mag. **40**, 36 (1949); Phil. Mag. **40**, 53 (1949); Phil. Mag. **40**, 929 (1949); Curran, Cockroft, and Insch, Proc. Phys. Soc. (London) **63**, 845 (1950).

[30] Good, Peaslee, and Deutsch, Phys. Rev. **69**, 313 (1946).

[31] Bradt, Gugelot, Huber, Medicus, Preiswerk, Scherrer, and Steffen, Helv. Phys. Acta **18**, 351 (1945); **19**, 77 (1946); **20**, 153 (1946); Phys. Rev. **68**, 57 (1945).

singly will detect both annihilation and nuclear gamma-radiations emitted from the source. When the angle between the counters, as measured at the source, is other than 180° coincidence counting detects only annihilation-nuclear and nuclear-nuclear (if there are two or more gamma-rays in cascade) coincidences. However, at an angle of 180° annihilation-nuclear, nuclear-nuclear and annihilation-annihilation coincidences are observed.

The difference between the coincidence rates when the counters form an angle of 90° with each other at the source and when they form a similar angle of 180° is a measure of the annihilation radiation only. If one calls this difference the net coincidence rate C_a, then the net coincidence rate per annihilation quantum recorded in a single counter becomes $C_a/N_a=\eta_a$, where η_a is now called the intrinsic efficiency for detecting annihilation radiation.

The singles counting rate, however, gives a value N, recording single counts for both annihilation radiation and nuclear gamma-rays. The net coincidence rate per recorded count becomes $C_a/N=A\eta_a$, A being $N_a/N=[e_\gamma(K+\beta^+)+2e_a\beta^+]/2e_a\beta^+$, from which can be determined the ratio K/β^+ if one knows e_γ, the counter efficiency for detecting the gamma-ray and e_a, the efficiency for detecting annihilation radiation.

To calibrate the efficiency of the counters, sources having a single gamma-ray of known energy and intensity were used.[32] In this way the authors were able to draw up a curve of counter efficiency *vs* gamma-ray energy using the same or similar counters in a similar geometrical arrangement as used in their K-capture/positron ratio experiments.

The quantity C_a/N is determined directly from the experiment. To determine η_a one needs a radioactive nucleus having no nuclear gamma-ray such that the experimentally determined value of C_a/N becomes in reality C_a/N_a. For this purpose, Cu^{64} was used. Although Cu^{64} has since been found[33] to emit a nuclear gamma-ray, the abundance of this gamma-ray is so low that its effect on the results probably lies within the

[32] Roberts, Elliott, Downing, Peacock, and Deutsch, Phys. Rev. **64**, 268 (1943).

[33] Bradt, Gugelot, Huber, Medicus, Preiswerk, Scherrer, and Steffen, Helv. Phys. Acta **19**, 219 (1946); M. Deutsch, Phys. Rev. **72**, 729 (1947).

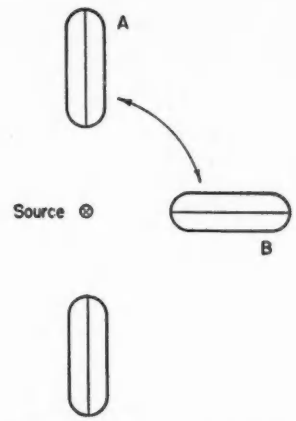

FIG. 5. Arrangement of counters used at MIT (see reference 30) for the determination fo the relative numbers of annihilation quanta and nuclear gamma-rays. Whereas the upper counter will detect annihilation-annihilation coincidences in position A it will not do so in position B.

errors of the experiment. This group[30] determined the K-capture/positron ratios for four isotopes—Na^{22}, V^{48}, Mn^{52} and Co^{58}. Their results are listed in Table I.

Both of these methods have provided reliable results by eliminating the necessity of direct measurement of the low energy radiations (x-rays or Auger electrons). A reliable determination of the intensities of the higher energy internal conversion electrons or gamma-rays is much less difficult.

A third indirect measurement of K-capture/positron ratios has been developed by Reynolds.[34] As in the case of the other indirect methods these results are probably somewhat more reliable than the direct methods but again can be applied only to a limited number of isotopes. The chief requirement for the technique to be applicable is that the radioactive isotope in question decays to either of two adjacent stable isobars.

The most thoroughly studied isotope of this type is Cu^{64}. Reynolds has investigated this isotope and his method will be described in terms of it. The technique is, however, just as applicable to other similar isotopes.

In order to produce a sufficient quantity of Cu^{64} gram quantities of copper were exposed to

[34] J. H. Reynolds, Phys. Rev. **79**, 789 (1950).

prolonged slow neutron bombardments in the Argonne laboratory pile. Since stable copper consists of Cu^{63} and Cu^{65}, both Cu^{64} and Cu^{66} will be produced by the (n, γ) process. After the long bombardment the copper was removed and allowed to decay until inappreciable amounts of radioactivity remained. This means that the Cu^{64} had decayed to Ni^{64} by K-capture and positron emission and to Zn^{64} by negatron emission and that the Cu^{66} had decayed to Zn^{66}. To this were added known (weighed) quantities of zinc and nickel. The three parts were thoroughly mixed. Since stable nickel contains isotopes at mass numbers 58, 60, 61, and 62 as well as 64, a mass spectrometric comparison of the nickel removed from the mixture and of a sample of the nickel used for preparing the mixture will measure the increase in the amount of Ni^{64} relative to the total quantity of added nickel. A similar measure may be made of the increase of Zn^{64} and Zn^{66} relative to Zn^{67} and Zn^{68}. If we call the increase in Ni^{64}, ΔNi^{64}, and the increase in Zn^{64}, ΔZn^{64}, then the ratio $\Delta Ni^{64}/\Delta Zn^{64} = (\beta^+ + K)/\beta^-$. The ratio β^-/β^+ is known.[27] From these measured quantities one finds a value for K/β^+. Reynold's value may be compared in Table I with other experimentally determined values of K/β^+ for Cu^{64}.

VII. Summary

The process of orbital electron capture by the nucleus has been discussed and analogies have been drawn between it and the process of positive beta-decay. Methods which are used for the detection of orbital electron capture have also been discussed. Those experiments which have led to the most accurate determinations of the relative intensities of the orbital electron capture and positron decay processes have been listed both in the text and in Table I. A few values of K/β^+ for other radioactive isotopes have been given in the literature. In general these values are only of a secondary nature in the experiment reported and result merely in an order-of-magnitude determination of the ratio.

The relatively few accurate measurements of the ratio K/β^+ which are available appear to be in agreement with the Fermi theory of beta-decay. At least one can say that, of the ratios which have been experimentally determined to date, none, with the possible exception of Na^{22}, differ greatly from the value predicted by theory (as is indicated in Table I).

The author expresses his thanks to Dr. George E. Owen for his helpful comments and suggestions on the manuscript and to Mr. Edward J. Clarkin for his help in preparing the line drawings.

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list (Am. J. Phys. **18**, 585 (1950)).

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A Laboratory Exercise in Nuclear Emulsion Technique

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A laboratory exercise in photographic emulsion track measurement is described, adaptable to any student level from the sophomore to the first-year graduate. The prime objective is student observation of emulsion tracks, and the unique pedagogical advantages of this recording medium are considered. Details of procedure and analysis of data are discussed.

THE object of the experiment here described is to bring into the undergraduate laboratory one of the important modern techniques of nuclear measurement, in a form within the reach of every physics department. The experiment is particularly elastic and may be adapted to any level from the sophomore to the first-year graduate. The photographic track method employed has a substantial pedagogic value, since the emulsion image constitutes a direct and complete image of the original nuclear event. The consequent pictorial quality has a strong student appeal. This experiment has been used with apparent success in the laboratory of an atomic physics course (junior year) at Brown University.

The photographic plate method is based on the fact that the action of light in rendering emulsion grains developable is duplicated by the passage of a charged (ionizing) nuclear particle. The path of such a particle through an emulsion appears in the developed image as a linear sequence of silver grains, called the track of the particle. Microscopic study of the track allows a determination of the particle's direction of motion, as well as the length of its path (the "range in emulsion"). The latter measurement is sufficient to determine the energy of the particle, since a unique and monotonic relation exists between the two. Before such use can be made of a particular emulsion, however, it must be "calibrated" by establishing empirically its range-energy relation for the particles to be studied.

Since the advent on the commercial market of specifically designed nuclear particle emulsions (about 1945), the microscopic-photographic technique has increased rapidly in stature among devices for the study of nuclear radiations. At the present time it is indispensable in cosmic-ray and fast neutron work, and is widely used in a diversity of experimental areas requiring the careful and accurate study of charged nuclear particles, from fission fragments to the weakest radiations from radioactive tracers. In the direct, detailed and permanent recording of a nuclear event, this device is rivalled only by the Wilson cloud chamber. The analogy between these two is in fact very close and may be exploited as a teaching aid. The emulsion technique differs mainly in its continuous sensitivity (no dead time) and large effective recording volume.[1]

The fundamental premise of the present exercise is that this tool has a place in the undergraduate laboratory, along with the visual scintillation apparatus and Geiger counter which have traditionally represented nuclear instrumentation. The photographic technique is considerably less subjective than the first and less costly than the second. As is common in the exploitation of these methods, the need for an electronuclear machine is obviated by the use of natural radioactivity, and that in a very weak and readily available form. In addition to introducing one of the newest nuclear tools, this experiment may be said to reintroduce the precision compound microscope to the undergraduate physics laboratory.

In brief, the experiment consists of track

[1] The linear scale of a recorded event is determined by the mean separation of atoms, and hence by the density as well as atomic stopping power of the recording medium. Thus the volume of normal air which is equivalent in stopping power to a conventional emulsion (1 inch $\times$ 3 inch $\times$ 100 micron) is roughly $50 \times 150 \times 0.2$ meter.

measurements in emulsions impregnated with a dilute solution of thorium nitrate. The tracks of alpha-particles ejected successively from isolated thorium nuclei form distinctive "stars" in the plates, after storage and later processing. The measurement and analysis of these tracks serves to determine both the factor of emulsion shrinkage during processing and the range of alpha-particles (of known energy) in the emulsion. The latter measurement constitutes a calibration of the emulsion for alpha-particles, expressed in terms of the emulsion "stopping power."[2] The content of the experiment is variable in the portion of the initial preparation and the amount of subsequent analysis assigned to the student, as indicated in detail in the sections which follow.

Apparatus

The major piece of equipment required is a high power microscope with oil immersion, at roughly 1000× magnification. A binocular eyepiece is convenient but not indispensable. Such microscopes are standard equipment in biological laboratories. The focal depth of such an instrument is usually sufficient (approximately one micron) for the accurate determination of vertical displacements. For this same purpose, the fine vertical adjustment should be calibrated in microns, as is common on such microscopes.

The photographic plates employed are available (directly from the manufacturer)[3] in the size of conventional microscope slides (1 inch ×3 inches). Emulsion thicknesses vary from 20 to 200 microns. The thinner grades (up to 50 microns) are more easily handled and are recommended for this experiment, since the thick varieties are necessary only in low intensity work, for adequate track yield. In all grades but the thickest, the plates to be mentioned cost roughly five dollars per dozen at present. Each of the two major commercial producers offers emulsions of various sensitivities, as described by Webb.[4] For this experiment the Eastman NTB or Ilford C-2 emulsions are satisfactory; these are the conventional varieties for proton range measurement. With these plates, tracks of electrons and of mesons will usually be observed along with those of alpha-particles, although these may be recorded with greater clarity in emulsions of maximum sensitivity (Eastman NTB-3 or Ilford G-5). Such extra phenomena should, of course, prove stimulating to the more inquiring students. This feature may be extended, if desired, by pre-exposing the plates to the neutron flux surrounding any available electronuclear machine. The resulting recoil proton tracks can be distinguished from those of alpha-particles in several ways: by their length, usually much in excess of the greatest natural alpha-ranges; by the fact that they always occur singly, whereas alpha-tracks may occur singly or in stars; and, with care in developing,[4] by their grain spacing.

Preparation

Three operations necessarily precede the track measurements themselves. Each of these may be assigned as part of the experiment or done by the instructor in advance, depending on the level of experimentation desired. The first is calibration of the track measuring device. This is conventionally a linear scale ruled on a glass reticle and inserted in the microscope eyepiece, and is calibrated *in situ* against an object of known size on the microscope stage. The latter is usually another ruled scale, of known (absolute) size, a "stage micrometer."

The initial stage of the experiment requires soaking the plates in thorium nitrate solution, followed by open-air drying and storage for a sufficient period to allow a reasonable number of thorium disintegrations to occur. Disposable variables are the concentration of thorium nitrate, time of soaking, and time of "exposure" (storage). These determine, in an integrated fashion, the concentration of stars, average number of prongs per star and the relative abundance of the various possible track lengths. It is not necessary to obtain an abundance of four- or five-pronged stars, for in practice track identification is by length rather than absolute prong number. Similarly, it is not necessary to "expose" for very long time to obtain an appreciable number of the desired ThC′ alpha-tracks,

[2] Ratio of air range to emulsion range, not atomic stopping power; not strongly dependent on energy (references 4, 9). See "Analysis."

[3] Eastman Kodak Company, Rochester, New York; Ilford Ltd., Ilford, London, England.

[4] Webb, Phys. Rev. **74**, 511 (1948).

since all of the radioactive disintegration products of thorium will accompany it into solution. Concentration and storage time have compensating effects, to a certain extent, but the storage time must be maintained long compared to the time required by the emulsion to dry after impregnation. Otherwise a significant number of stunted tracks will be observed, tracks recorded when the emulsion was wet and swollen. A convenient set of values follows: soaking in 0.1 percent thorium nitrate (by weight) for 10–30 minutes followed by storage for 2–3 weeks; or, for example, 1 percent thorium nitrate and storage for 2–3 days. The Wratten No. 2 light is safe for Eastman NTB and NTB-3 emulsions.

After storage the plates must be developed and fixed in the usual way. For emulsions of 50 microns or less the processing is straightforward —e.g., development for 2–10 minutes in undiluted D-19 followed by a few minutes in 2 percent acetic acid stop bath and fixation (for one hour or several, depending on thickness) in F-5 or 30 percent hypo (alternatives are mentioned by Webb).[4] A preliminary ten-minute soaking in water at 30°C aids the diffusion of solutions into the emulsion. Greater thicknesses require more elaborate procedures.[5] After washing, plates are dried slowly (as in an enclosed air volume) and are ready for observation.

Measurements

The successive alpha-particles emitted by a thorium nucleus (and products) differ substantially in energy, and the components of any star should in principle be identifiable on this basis alone, assuming a monotonic relation between energy and emulsion range. Actually, this experiment calls only for identifying the ThC′ alpha-particles. Such identification is a simple matter, since that particle is fully 2 Mev more energetic than any of its companions.[6] The ThC′ tracks only are selected for measurement, so that the particles may be assumed homogeneous in initial energy.

It is necessary that every measured track should lie entirely within the emulsion, which may readily be checked by noting the presence of single grains in the random background lying both above and below each end of each track used. The projection on the focal plane (horizontal) is measured by reference to the eyepiece scale whose calibration has been discussed. The vertical projection of each track is determined by the fine focus control, as the distance moved in shifting the focus from one terminal grain to that on the opposite end. If the usual precaution against backlash is observed, this measurement is reproducible within a micron. Such measurements are performed for a number of ThC′ tracks sufficient to guarantee statistical significance—possibly twenty to fifty tracks, depending again on the student level involved.

For an average undergraduate a single afternoon (three hours) will usually suffice for the minimum of track measurements, including time taken to learn the technique. Two afternoons will, in general, be ample for fifty track measurements and the preliminary eyepiece scale calibration to be done by the student. If each prepares and develops his own plates another hour or two will probably be added to the time expended.

Analysis

Two unknown parameters are to be determined. The shrinkage factor a is the ratio of emulsion thickness before processing to that in the final (measured) form. This factor is roughly two, corresponding to the removal during fixation of the great bulk of light-sensitive salt. The shrinkage occurs almost wholly in a direction normal to the glass backing,[7] and is therefore representable by a linear dimension ratio, as just assumed. The second parameter R to be determined is the range in the emulsion of these alpha-particles. From such a value the emulsion stopping power[2] may be obtained directly, since the air range is known.[8] Both parameters may be determined from the length measurements (horizontal and vertical components of each track, called p and z, respectively).

[4] Wilson and Vanselow, Phys. Rev. **75**, 1144 (1949).

[5] For thorium series alpha-energies, see a standard atomic physics text; recent values can be found in W. H. Sullivan *Trilinear Chart of Nuclear Species* (John Wiley and Sons, Inc., New York, 1949).

[7] Powell, Occhialini, Livesey, and Chilton, J. Sci. Instr. **23**, 102 (1946).

[8] ThC′ alpha energy = 8.77 Mev (reference 6). The range *vs* energy curve for alpha-particles in normal air is given by Bethe and Livingston, Revs. Modern Phys. **9**, 245 (1937), Fig. 29.

In the original emulsion (unprocessed), all tracks of ThC′ alpha-particles should have the same length, *viz.*,

$$R^2 = p^2 + z_0^2,$$

where z_0 is the vertical component. In the processed and dried emulsion however, the track lengths will differ, depending on track orientation because of the vertical shrinkage; i.e.,

$$R^2 = p^2 + z^2 a^2.$$

This is the observational equation corresponding to the above, with the substitution $z_0 = za$; the vertical projection actually measured (in the dried emulsion) is z.

The most satisfactory way to obtain the desired parameters from the data is the least squares determination. Although this is included as an appendix, it is perfectly straightforward and advanced students may reasonably be required to work it out. A less rigorous procedure is the following: Divide the track measurements into two groups, according to the relative magnitudes of horizontal and vertical components. Assuming a value (say 2) for the shrinking factor, determine from the more nearly horizontal track group a tentative R value, which is then used to determine a new a from the remaining data. In this way a successive approximation method is used. The simplest approach, recommended for elementary students, is to give for a a value previously determined by the instructor, and require only the determination of the emulsion range R. These values may be checked against the published literature;[4,9] agreement within a few percent may reasonably be anticipated.

Two direct extensions are possible. First, the procedure may be repeated for other tracks than the longest (ThC′) variety. In this way a curve may be plotted of emulsion stopping power[2] as a function of energy, whose shape is of practical and theoretical interest.[4,10] Such a procedure entails greater difficulty than might at first be supposed, for the identification of the various alpha-particle tracks from length alone requires in some cases a high degree of accuracy in length measurements. The small differences between various thorium series alpha-ranges, their "straggling" (statistical fluctuations in range) and the limited depth resolution of the microscope combine to make this extension considerably more trying than the main portion of the experiment. Second, the stopping power obtained may be checked by an absolute calculation from measured values of atomic stopping power for the constituent atoms.[8,11] Such a calculation may correlate closely with the content of the usual atomic or nuclear physics course.

[9] Lattes, Fowler and Cüer, Proc. Phys. Soc. (London) **59**, 883 (1947).

[10] Peck, Phys. Rev. **72**, 1121 (1947).

References

For orientation in any such project, students should certainly have access to Powell's splendid book[12] of emulsion photomicrographs of a great variety of nuclear particle tracks. A discussion of the experimental phases directly involved here is given by Yagoda,[13] in addition to useful historical material. Additional experimental and historical material is available in various survey articles.[4,9,14,15] Finally the existence of a laboratory manual[16] pertaining to nuclear photographic technique may be noted, though it is in no way necessary to the exercise described in this article.

Appendix: Least Squares Determination of Parameters

Parameters:

a = (thickness of unprocessed emulsion)/(thickness of processed emulsion),

R = range of alpha-particles in emulsion (monoenergetic group).

Data:

p_i = horizontal projection of ith track, in microns,

z_i = vertical projection of ith track, in microns.

Observational equations:

$$p_i^2 + a^2 z_i^2 - R^2 = v_i \text{ (residual)},$$

$(i = 1 \cdots N$, number of tracks measured).

[11] J. B. Hoag, *Electron and Nuclear Physics* (D. Van Nostrand Company, Inc., New York, 1945).

[12] C. F. Powell and G. P. S. Occhialini, *Nuclear Physics in Photographs* (Oxford University Press, London, 1947).

[13] H. Yagoda, *Radioactive Measurements With Nuclear Emulsions* (John Wiley and Sons, Inc., New York, 1949), Chapters 1, 12.

[14] H. Yagoda, Nucleonics **2**, No. 5, 2 (1948).

[15] M. Shapiro, Revs. Modern Phys. **13**, 58 (1941).

[16] *Nuclear Physics Laboratory Manual* (Heinecke Instrument Corporation, Rochester, New York).

Normal equations:

$$\sum_i v_i \frac{\partial v_i}{\partial a^2} = \sum_i (p_i^2 + a^2 z_i^2 - R^2) z_i^2 = 0,$$

$$\sum_i v_i \frac{\partial v_i}{\partial R^2} = \sum_i (p_i^2 + a^2 z_i^2 - R^2) = 0;$$

whence

$$a^2 \sum_i z_i^4 - R^2 \sum_i z_i^2 + \sum_i p_i^2 z_i^2 = 0,$$

$$a^2 \sum_i z_i^2 - R^2 N + \sum_i p_i^2 = 0.$$

After numerical substitutions from the data, these equations may be solved for a^2 and R^2.

Motion Relative to the Surface of the Rotating Earth

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When an observer moves over the face of the earth, there is a change in both the magnitude and the direction of the apparent acceleration due to gravity. At speeds now becoming attainable, the change is just about observable with modern aircraft navigating instruments. The change consists of two terms—a Coriolis term and a centripetal term. If a particle could be dropped through a vacuum from an aircraft flying a great circle course, the Coriolis deflection of the falling particle, *as seen by the airman*, would be independent of the aircraft's velocity.

AT aircraft speeds hitherto attainable, not much interest was attached to some of the terms in the well-known expression, Eq. (11) below, for motion relative to rotating coordinates, but at the speeds now possible, these terms acquire real interest and significance. In the present state of development of navigating instruments, the uncertainty or error with which automatic pilots and stabilizers operate is just about equal to the magnitude of some of the effects to be described. Thus, further increases in aircraft speeds, accompanied by further improvements in aircraft instruments, should result in the occurrence of situations in which some of these effects will be observable.

Notation

$\mathbf{r}$ —radius vector from the center of the earth to any point on the surface of the earth;
ϕ —angle between the equatorial plane and $\mathbf{r}$, called the latitude;
$\mathbf{r}_0$ —unit vector in the direction of $\mathbf{r}$;
$\boldsymbol{\Theta}$ —unit vector directed eastward;
$\boldsymbol{\Phi}$ —unit vector directed northward; $\mathbf{r}_0$, $\boldsymbol{\Theta}$, and $\boldsymbol{\Phi}$, constitute a right-handed set of unit vectors in the spherical polar coordinate system whose polar axis is the earth's polar axis, and whose coordinate angles are longitude, measured eastward, and latitude;
ω —angular rate of the earth on its polar axis;
$\mathbf{v}_a$ —velocity of an aircraft, as seen by an observer fixed on the earth;
$\dot{\mathbf{v}}_a$ —acceleration of the aircraft, as seen by the observer fixed on the earth;
$\mathbf{v}_p$ —velocity of a projectile, as seen by the observer fixed on the earth;
$\dot{\mathbf{v}}_p$ —acceleration of the projectile, as seen by the observer fixed on the earth;
$\mathbf{u}_a$, $\dot{\mathbf{u}}_a$, $\mathbf{u}_p$, and $\dot{\mathbf{u}}_p$—the same velocities and accelerations as seen by an observer in inertial space;
$\mathbf{G}$ —the earth's gravitational field, or the gravitational vector;
$\mathbf{g}$ —the acceleration caused by gravity, or the gravity vector as seen by the observer fixed on the earth;
$\mathbf{f}$ —the gravity vector as it appears to an observer moving on the face of the earth;
α —angle between $\mathbf{g}$ and $\mathbf{G}$;
β —angle between $\mathbf{g}$ and $\mathbf{r}$;
γ —angle between $\mathbf{G}$ and $\mathbf{r}$;
δ —angle between $\mathbf{f}$ and $\mathbf{g}$;
$\boldsymbol{\delta}$ —vector, having magnitude δ, and indicating the direction of the axis about which δ is measured;

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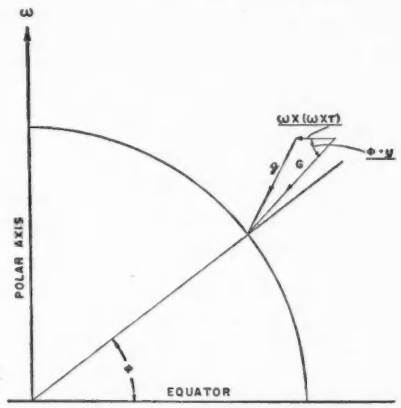

FIG. 1. Diagram to illustrate Eqs. (1) and (2). In this figure **G** is the gravitational vector; **g** is the gravity vector; $\omega\times(\omega\times\mathbf{r})$ is the centripetal acceleration of a point at the latitude ϕ on the earth's surface; α is the angle between **g** and **G**; γ is the angle between **G** and **r**; $(\phi+\gamma)$ is the angle between **G** and $\omega\times(\omega\times\mathbf{r})$.

ϵ —angle subtended by the Coriolis deflection of a body dropped from the height H above the surface of the earth;

$\boldsymbol{\varepsilon}$ —vector whose direction is that of the axis about which ϵ is measured;

ζ —deflection of the dropped body as seen by an observer whose velocity is equal to the horizontal component of $\mathbf{v}_p$;

$\boldsymbol{\zeta}$ —vector whose direction is that of the axis about which ζ is measured.

The term "tension" will be used to mean the tension per unit mass of the bob, in a string supporting a pendulum; it is therefore a quantity having the same dimensions as acceleration.

The Plumb Line, or Static Vertical

Suppose first that the earth, considered as a uniform sphere, stood still in inertial space; i.e., that it did not rotate on its polar axis. Then, at every point on the earth's surface, there would be observed a gravitatational field (or acceleration) along the earth's radius at that point. Let this acceleration be denoted by **G**. If the earth did not rotate, the surface of a pond or ocean would, at all points, be perpendicular to **G**, and a simple pendulum would hang along **G**.

Now, let us find the direction in which a simple pendulum (plumb bob) hangs on the rotating earth. Any point constrained to stay fixed with respect to the earth's surface is undergoing a centripetal acceleration towards the earth's polar axis (*not* towards the earth's center, except at the equator). This centripetal acceleration is equal in magnitude to $\omega^2 r\cos\phi$, where ϕ is the latitude of the point.

Thus, the pendulum's support experiences a force towards the earth's polar axis, while the earth's gravitational field is pulling the bob in the direction of **G**. The resultant "tension" in the string supporting the bob is along the difference between the two force vectors: this resultant vector is called the acceleration due to gravity; its direction is called the plumb line, or static vertical. In the form of an equation,

$$\mathbf{g}=\mathbf{G}-\omega\times(\omega\times\mathbf{r}). \tag{1}$$

The situation is shown pictorially, on an exaggerated scale, in Fig. 1. If we apply the law of cosines and the law of sines to the triangle formed by the vectors **g**, **G**, and $\omega\times(\omega\times\mathbf{r})$, we obtain the relations

$$g^2=G^2-2\omega^2 rG\cos\phi\cos(\phi+\gamma)+(\omega^2 r\cos\phi)^2;$$
$$\sin\alpha=\omega^2 r\sin(\phi+\gamma)\cos\phi/g.$$

For all practical purposes, these equations reduce to:

$$g=G-\omega^2 r\cos^2\phi; \tag{2}$$
$$\alpha=\omega^2 r\sin\phi\cos\phi/g.$$

Now, the surface of the earth is perpendicular to **g**, not to **r** nor to **G**. The figure of the earth is therefore, approximately, an ellipsoid of revolution, whose dimensions are as follows:

equatorial radius, $a=3963.34$ mi. $=20{,}926{,}000$ ft;
polar radius, $b=3949.99$ mi. $=20{,}856{,}000$ ft;
$a-b=13.35$ mi. $=70{,}000$ ft.

The angle β between the radius vector and the normal of an ellipse is given by the equation:

$$\tan\beta=(a-b)\sin\phi\cos\phi/(a\sin^2\phi+b\cos^2\phi).$$

Since a and b are so nearly alike, and since β is a very small angle, this equation may be replaced by the more convenient one:

$$\beta=(a-b)\sin\phi\cos\phi/r. \tag{3}$$

Combining Eq. (3) with the second of Eq. (2), we obtain for the angle between **G** and **r**:

$$\gamma=\alpha-\beta=(\omega^2 r/g-(a-b)/r)\sin\phi\cos\phi. \tag{4}$$

Now, $\omega^2 r/g = 0.00347$; $(a-b)/r = 0.00337$. Thus, γ is quite small compared to α or to β; it will be neglected in what follows.

Experimentally, it is found that g may be represented, with an accuracy of one part in 100,000 or so, by the formula:[1]

$$\left.\begin{aligned} g &= (32.2577 - 0.1699\cos^2\phi)\ \text{ft/sec}^2; \\ \mathbf{g} &= -(32.2577 - 0.1699\cos^2\phi)\mathbf{r}_0 \\ &\quad + 0.1112\sin\phi\cos\phi\boldsymbol{\Phi}. \end{aligned}\right\} \quad (5)$$

Combining Eq. (5) with the first of Eq. (2), we have

$$\left.\begin{aligned} G &= (32.2577 - 0.0587\cos^2\phi)\ \text{ft/sec}^2, \\ \mathbf{G} &= -(32.2577 - 0.0587\cos^2\phi)\mathbf{r}_0. \end{aligned}\right\} \quad (6)$$

It should be noted, in passing, that Eqs. (4) and (6) are not in exact agreement with those[2] to be derived from simple potential theory for a homogeneous oblate spheroid.

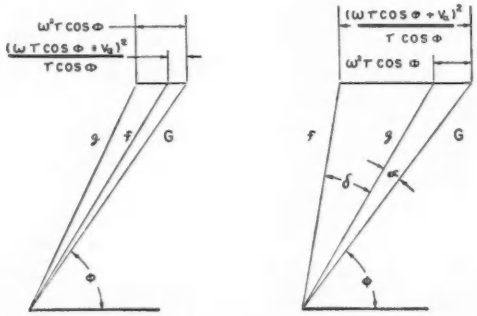

FIG. 2. Two diagrams to illustrate the relation between the plumb line g and the dynamic vertical f for an aircraft travelling straight east or straight west along a parallel of latitude. For the westbound aircraft, v_a is supposed to be almost, but not quite, equal to $-\omega r\cos\phi$, so that $(\omega r\cos\phi + v_a)$ is but a small fraction of $\omega r\cos\phi$ itself, and δ is approximately equal to $-\alpha$. In the eastbound example, v_a is supposed to be nearly equal to $+\omega r\cos\phi$, so that δ is equal to something less than 4α.

[1] This formula is obtained from the data on p. 2675, *Handbook of Physics and Chemistry*, 31st Edition (Chemical Rubber Publishing Company, 1949). The numerical values of the coefficients in Helmert's formula, p. 2358, are based on older data. Equation (5) is thus the same as Helmert's, except for the omission of higher order terms in the latitude, and for the omission of the term for the variation of g with altitude above the face of the earth.

[2] The calculation of the potential of a homogeneous oblate spheroid is presented as an exercise by W. E. Byerly, *An Elementary Treatise on Fourier's Series, etc.*, (Ginn and Company, New York, 1893), p. 157, problem 3. If one now computes the gradient of this potential, he will find that the results thus obtained are in qualitative, but not exact, agreement with Eqs. (4) and (6). This point is discussed by Sigmund Hammer, Geophysics **8**, 57 (1943).

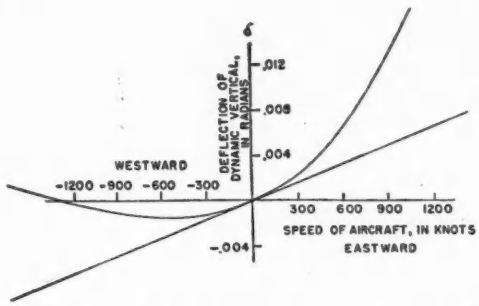

FIG. 3. The deflection δ of the dynamic vertical *vs* speed of the aircraft when $\phi = 45°\text{N}$. The parabola is the graph of Eq. (10), for an aircraft flying straight east or west along the 45th parallel; the straight line is the graph of Eq. (14), for an aircraft crossing this parallel along the arc of any intersecting great circle, or at the instant of tangency for an aircraft flying along a great circle which is tangent to this parallel at its northernmost point.

The Dynamic Vertical

Now, let us consider a simple pendulum being carried in a vehicle moving with constant speed v_a along an arc of the earth's surface. If the motion be along the equator, the expression for the centripetal acceleration of the pendulum's support becomes:

$$\begin{aligned} -u_a^2/r &= -(\omega r + v_a)^2/r \\ &= -(\omega^2 r + v_a^2/r + 2\omega v_a). \end{aligned} \quad (7)$$

In this expression $-\omega^2 r$ is the centripetal acceleration of a point fixed on the earth's equator, $-v_a^2/r$ is the centripetal acceleration of the vehicle with respect to any set of axes fixed in the rotating earth, and $-2\omega v_a$ is the compound acceleration, or the acceleration of Coriolis. This is probably the simplest example or illustration of a Coriolis acceleration.

To the observer in the aircraft, the "tension" of the pendulum's string appears to be changed by the amount $-(v_a^2/r + 2\omega v_a)$; or the apparent acceleration of gravity becomes

$$f = g - (v_a^2/r + 2\omega v_a). \quad (8)$$

If the aircraft now move along any other parallel of latitude, Eq. (7) becomes:

$$\begin{aligned} -\frac{u_a^2}{r\cos\phi} &= -\frac{(\omega r\cos\phi + v_a)^2}{r\cos\phi} \\ &= -(\omega^2 r\cos\phi + v_a^2/r\cos\phi + 2\omega v_a). \end{aligned} \quad (9)$$

The reader can easily identify or interpret the terms in this expression for himself. If we now apply the law of cosines and the law of sines to the triangle formed by f, G, and $u_a^2/r\cos\phi$ as shown in Fig. 2, we obtain the relations

$$f^2=G^2-\frac{2G(\omega r\cos\phi+v_a)^2}{r}+\frac{(\omega r\cos\phi+v_a)^4}{(r\cos\phi)^2}$$

$$\sin(\alpha+\delta)=(\omega r\cos\phi+v_a)^2\tan\phi/rf.$$

For any foreseeable value of v_a along any parallel of latitude in the temperate or torrid zones, these relations reduce to:

$$\left.\begin{aligned} f&=G-(\omega r\cos\phi+v_a)^2/r\\ &=g-(v_a^2/r+2\omega v_a\cos\phi)\\ \delta&=(v_a^2/r\cos\phi+2\omega v_a)\sin\phi/f\\ &=v_a(v_a\tan\phi+2\omega r\sin\phi)/rf.\end{aligned}\right\}\quad(10)$$

The direction taken by f is called the dynamic vertical, and δ is the deflection of the dynamic vertical from the static vertical. The centripetal term $v_a^2\tan\phi/rf$ is always towards the equator; the Coriolis term $2\omega v_a\sin\phi/f$ is always towards the pilot's right in the northern hemisphere, and towards his left in the southern hemisphere. A graph of δ *vs* v_a is shown in Fig. 3.

It is sometimes asserted that the difference between f and g is just the Coriolis acceleration. When v_a is small compared to $\omega r\cos\phi$, this is quite true; when $v_a=-\omega r\cos\phi$, the aircraft is standing still in inertial space (neglecting the acceleration of the earth in its orbit around the sun), then $f=G$, and

$$\delta=-\alpha=\omega v_a\sin\phi/f,$$

which is half the Coriolis term. Similarly, if the aircraft flies westward with such a speed that its centripetal acceleration in inertial space is equal to that of a point at rest on the earth at that latitude, then $v_a=-2\omega r\cos\phi$, $f=g$, and $\delta=0$; the Coriolis term is equal and opposite to the centripetal term in the expression for the deflection of the dynamic vertical.

Equations (7), (8), (9), and (10), are but particular examples of the general expressions below. If the aircraft flies with velocity $\mathbf{v}_a$ along any arc of the earth's surface, its acceleration with respect to inertial space is given by the well-known equation:

$$\dot{\mathbf{u}}_a=\dot{\mathbf{v}}_a+2\boldsymbol{\omega}\times\mathbf{v}_a+\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}).\quad(11)$$

If a pendulum be hung in this airplane, the "tension" will be given by this expression:

$$\mathbf{f}=\mathbf{G}-\dot{\mathbf{u}}_a=\mathbf{g}-(\dot{\mathbf{v}}_a+2\boldsymbol{\omega}\times\mathbf{v}_a),\quad(12)$$

and the angle between $\mathbf{f}$ and $\mathbf{g}$ may be obtained from the relation

$$\sin\delta=|\mathbf{f}\times\mathbf{g}|/fg=|(\dot{\mathbf{v}}_a+2\boldsymbol{\omega}\times\mathbf{v}_a)\times\mathbf{g}|/fg.$$

Since f and g are very nearly equal in magnitude, and since δ is a very small angle, this reduces to:

$$\boldsymbol{\delta}=\mathbf{g}\times(\dot{\mathbf{v}}_a+2\boldsymbol{\omega}\times\mathbf{v}_a)/g^2.\quad(13)$$

Equation (13) is a good enough approximation for $\boldsymbol{\delta}$ for any flight along a great circle on the earth's surface, or along parallels of latitude in the temperate or torrid zones; but not along any course in which the bank angle exceeds 10° or so.

Of course, on the surface of an ellipsoid of revolution, there is no such thing as a true "great circle" other than the equator. However, for our purpose, we may consider a path in which $\dot{\mathbf{v}}_a$ is parallel to $\mathbf{g}$ as comparable to a great circle course on a sphere; for such a course,

$$\boldsymbol{\delta}=2\boldsymbol{\omega}\mathbf{v}_a\sin\phi/g.\quad(14)$$

Thus, for a great circle course, the statement, that the deflection of the dynamic vertical is just the Coriolis deflection, is quite correct. A graph of Eq. (14) is shown in Fig. 3.

The deflection of the dynamic vertical is of interest because the aircraft's automatic stabilizers adjust or balance the ship with respect to the dynamic vertical, not with respect to the static vertical, whereas, in celestial navigation, for example, one desires to know the angle between line of sight to a star and the static vertical, not between line of sight and dynamic vertical.

The Trajectory in Vacuum

In 1943, Kirkpatrick[3] pointed out that even for so short a range as that of the broad jump, the "trajectory" of the athlete jumping eastward at the equator is measurably greater than that of the athlete jumping westward—by about $\frac{1}{16}$ inch. For the well-thrown discus, the difference is about one inch; for the long range artillery projectile, the difference is proportionately still greater.

[3] P. Kirkpatrick, Am. J. Phys. **11**, 303 (1943); **12**, 7 (1944).

The discussion can be rendered both simple and vivid by considering a bomb dropped from a balloon at rest at the altitude H above the earth's surface at the equator. To the observer in inertial space, the velocity of the bomb, when it is released from the balloon, is given by:

$$\mathbf{u}_p=\boldsymbol{\omega}\times(\mathbf{r}+\mathbf{H}).$$

The bomb then travels through inertial space along the arc of the Kepler ellipse for which

$$\text{major axis}=\frac{r^2G(r+H)}{2r^2G-\omega^2(r+H)^3},$$

$$\text{minor axis}=\frac{\omega(r+H)^3}{[2r^2G(r+H)-\omega^2(r+H)^4]^{\frac{1}{2}}},$$

where G is the magnitude of the gravitational field at the surface of the earth, not at the altitude H.

To the observer on the earth, the bomb will appear to fall almost, but not quite, along the plumb line (or static vertical): more exactly, it will appear to fall east of the plumb line along the arc of the semicubic parabola,

$$x_E=2\omega(2H^3)^{\frac{1}{2}}/3g^{\frac{1}{2}}. \qquad (15)$$

Even this expression is not rigorously exact, but it is good enough for our purpose.

The quantity x_E is the Coriolis deflection of the falling bomb. Let the angle subtended by x_E at the height H be denoted by ϵ. Since ϵ is a very small angle, we may write

$$\epsilon=2\omega(2H)^{\frac{1}{2}}/3g^{\frac{1}{2}}. \qquad (16)$$

Similarly, if the bomb be dropped from an aircraft moving along the earth's equator with horizontal speed v_a, it will travel through inertial space along the arc of the Kepler ellipse for which

$$\text{major axis}=\frac{r^2G(r+H)}{2r^2G-(r+H)[v_a+\omega(r+H)]^2};$$

$$\text{minor axis}=\frac{(r+H)^2[v_a+\omega(r+H)]}{[2r^2G(r+H)-(r+H)^2[v_a+\omega(r+H)]^2]^{\frac{1}{2}}}.$$

To the observer on the earth, the path of the falling bomb will be very nearly the parabola

$$x=v_a(2H/g)^{\frac{1}{2}}.$$

The difference between the actual path and this parabola is given, with sufficient accuracy, by Eq. (15), above.

Equation (16) is but a particular case of the more general relation stated in Eq. (17), below. For the acceleration of a bomb falling freely through a vacuum,

$$\dot{\mathbf{u}}_p=\mathbf{G}=\dot{\mathbf{v}}_p+2\boldsymbol{\omega}\times\mathbf{v}_p+\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}). \quad (11, \text{repeated.})$$

Therefore,

$$\dot{\mathbf{v}}_p=\mathbf{g}-2\boldsymbol{\omega}\times\mathbf{v}_p;$$

and

$$\boldsymbol{\varepsilon}=\int\int\dot{\mathbf{v}}_p\times\mathbf{g}\,dt\,dt/gH$$

$$=2\int\int\mathbf{g}\times(\boldsymbol{\omega}\times\mathbf{v}_p)\,dt\,dt/gH.$$

Let $\mathbf{v}_p=\mathbf{v}_a+\mathbf{g}t$, where $\mathbf{v}_a$ is the horizontal component of $\mathbf{v}_p$ (a constant if atmospheric resistance and also higher order terms be neglected). Then

$$\boldsymbol{\varepsilon}=2\boldsymbol{\omega}[\mathbf{v}_a\sin\phi/g+(2H)^{\frac{1}{2}}\cos\phi\boldsymbol{\Phi}/3(g)^{\frac{1}{2}}] \qquad (17)$$

Thus, $\boldsymbol{\varepsilon}$ has two components, one proportional to v_a, and to the right of an observer on the ground, facing in the direction of v_a in the northern hemisphere (to his left in the southern hemisphere); the other proportional to $(H)^{\frac{1}{2}}$, and always to the east.

The observer in an aircraft, however, would measure the deflection of a bomb falling through a vacuum from his own dynamic vertical, not from the static vertical. If we denote this deflection by $\boldsymbol{\zeta}$, then

$$\boldsymbol{\zeta}=\boldsymbol{\varepsilon}-\boldsymbol{\delta}. \qquad (18)$$

Then, combining Eqs. (14), (17), and (18), we obtain

$$\boldsymbol{\zeta}=2\boldsymbol{\omega}\cos\phi\boldsymbol{\Phi}(2H)^{\frac{1}{2}}/3(g)^{\frac{1}{2}}-\mathbf{g}\times\dot{\mathbf{v}}_a/g^2. \qquad (19)$$

Now, $\mathbf{g}\times\dot{\mathbf{v}}_a=0$ on a great circle course, so that, in this case, $\boldsymbol{\zeta}$, as seen by the pilot of the aircraft, is equal to what the observer on the ground would see if the bomb had been dropped from a stationary balloon, namely, the Coriolis deflection of a falling object.

This article is the result of discussions with colleagues engaged in the design of aircraft instruments. The data for Fig. 3 were computed by Mr. William J. Portway. The three figures were drawn by Mr. Richard Kamholz.

NOTES AND DISCUSSION

A Driver for the Calthrop Resonance Pendulum

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J. E. CALTHROP has described a very interesting experiment with a resonance pendulum.[1] Calthrop's experiment provides a study of resonance and determines the damping coefficient in the appropriate equation of motion. In brief, the apparatus consists of a heavy compound pendulum which drives a much lighter approximately simple pendulum. The latter is supported from a gallows or rod mounted a few inches above the knife edge support. A disadvantage exists in that it is necessary to keep starting the heavy pendulum, since the amplitude dies down with time. To overcome this disadvantage a driving apparatus free from intermittent contacts has been devised. The driving apparatus makes use of the well-known principle that if an iron core is introduced into a solenoid which forms part of a series-resonant *LCR* circuit, the latter is thrown out of resonance, causing a large drop in the current. One has only to attach a short length of iron rod to the bottom of the heavy pendulum in such a way that the iron rod passes into the solenoid at the end of each pendumum swing as shown in Fig. 1. As the rod approaches the solenoid, the current is large. Hence, the iron rod is pulled in. As it goes in, the current falls to a very small value, with the result that the pull is reduced to such extent that the pendulum can swing back again. The pendulum thus receives a small net tug at the extreme end of each swing sufficient to maintain the motion.

Although this method of driving a pendulum has been known for a long time, I have nowhere seen a satisfactory theoretical explanation of the phenomenon. Neither I nor anyone I have talked with about this type of driver understands just why it operates as it does. Evidently, the force exerted on the iron rod must be a two-valued function of the position of the pendulum and is on the average greater when the rod moves into the solenoid than it is when the rod moves out. Otherwise, there would be no way to account for the fact that the magnetic field of the solenoid transfers energy to the pendulum. It is not easy to see why the force function is double-valued.

Values of the circuit parameters do not seem to be critical. Inductances between 0.5 and 1.0 henry (with the iron plunger out) with corresponding capacitances of approximately 14 and 7 microfarads are satisfactory. A current in the neighborhood of 1 amp at series resonance (iron plunger out) is satisfactory. This current value is obtained by choice of the proper tap on the auto transformer.

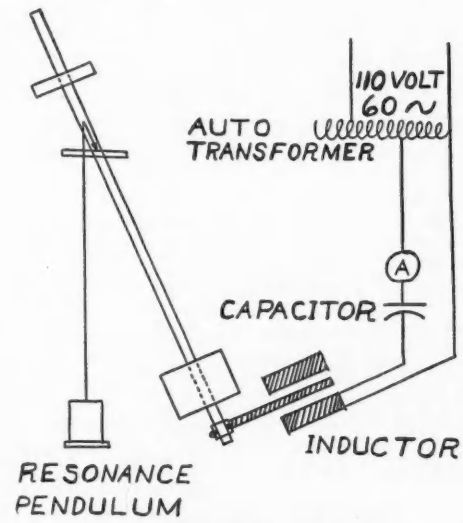

FIG. 1. Resonance-pendulum with driver.

[1] J. E. Calthrop, *Advanced Experiments in Practical Physics* (William Heinemann, Ltd., London, 1938), first edition, p. 14ff.

A Theorem on Deviation in Thin Lenses

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MOST elementary physics textbooks give derivations of the lens equation either by the method of change of curvature of the wave front or by application of theorems on similar triangles to a standard ray diagram. The first of these methods is very cumbersome for an elementary course; the second is hard to remember and gives little or no insight into lens action. It is possible to derive the lens equation quite simply, and also to obtain certain other results, by application of a theorem that appears not to be widely known, although it is implicit in all thin lens theory.

This theorem can be stated either for thin lenses or for prisms of small angle. For a small angle prism, it says that all rays passing through the prism at nearly normal incidence undergo the same deviation regardless of their direction. The stipulation of nearly normal incidence is to be understood as meaning that all the angles of incidence and refraction are so small that their sines can be replaced by the angles themselves without exceeding the allowable error. The theorem can be proved as follows.

Let the angles of incidence and refraction be designated as in Fig. 1, and let the index of the prism be n. Then, because of Snell's law and the assumed smallness of the angles, $r_1 = i_1/n$ and $r_2 = ni_2$. Now let i_1 be increased by a small angle θ; then r_1 will increase by θ/n, so that the ray inside the prism will turn through θ/n; therefore i_2 will decrease by θ/n; therefore, r_2 will decrease by θ. Since the incident and emergent rays have both turned through the same angle, θ, the deviation has not changed; and this proves the theorem.

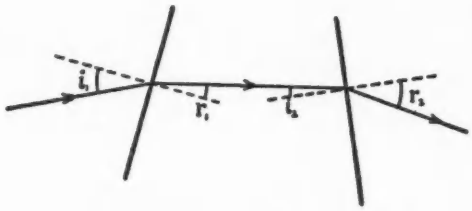

FIG. 1. Passage of ray through small angle prism.

Applied to a thin lens, the theorem states that all rays incident nearly normally *at a given point* of the lens undergo the same deviation. This follows at once from the preceding paragraph, since any small section of a lens can be approximated by a section of a prism. It is not true, of course, that rays incident at *different* points undergo the same deviation.

To derive the lens equation, consider first a ray parallel to the axis and incident at M (Fig. 2). The deviation angle

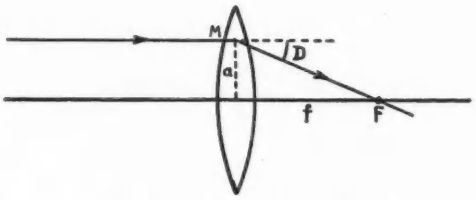

FIG. 2. Deviation and focal length.

D being small, is equal to a/f. Next let a ray from a point object O on the axis (Fig. 3) strike the lens at M; then the

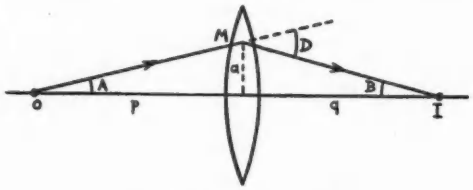

FIG. 3. Derivation of lens equation.

deviation D must be the same as in Fig. 2, according to the theorem. Since, in Fig. 3, $A=a/p$ and $B=a/q$, and since $D=A+B$, it follows that $a/f=(a/p)+(a/q)$, from which the lens equation is obtained by dividing out a.

The theorem on deviation also provides an easy way of seeing how the image moves if the object distance is changed. Suppose that in Fig. 3 the object is moved along the axis toward the lens; then the ray from O to the fixed point M will turn counterclockwise about M, and so the emergent ray must also turn counterclockwise about M since the deviation angle does not change; therefore the image, which is at the intersection of the emergent ray with the axis, will move away from the lens. If the object continues to approach the lens, the image will continue to recede until the object is at the principal focus and the emergent ray is parallel to the axis; and after that the image, now virtual, will move closer to the lens, as is easily seen by prolonging the emergent ray backward to the axis and noting that the intersection now moves *toward* the lens as the emergent ray turns counterclockwise. This method can readily be applied to diverging lenses, as well as to the lenses of a two-lens system.

In graphical work, the theorem makes it quite easy to trace any ray whatever through a thin lens, provided the incidence is nearly normal. All that is necessary is to draw a ray parallel to the axis and incident at the same point as the given ray; after emergence, this ray will go through the principal focus, and thus its deviation angle can be constructed. According to the theorem, this deviation angle is equal to the deviation angle of the given ray. Thus the direction of emergence of the given ray is determined, and can be drawn with the aid of a protractor, or, approximately, by eye. This construction is helpful in tracing rays through a two-lens system.

Physical Values which Depend on the Standard Deviation of Another Value

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TO determine the relation between two measurable physical quantities, say a and b, numerous measurements of a at a constant b are made and a mean value of a is calculated. When this procedure is repeated for several values of b, the function $a=f(b)$ can be established. Only *mean* values are represented in this equation. However, there are instances in which the value of a is more properly expressed as a function of the spread of a values at another b. Such an expression follows directly from the well-known probability theory of tensile strength and is thus not new; but apparently it has not been sufficiently emphasized. In the following it is derived for a special case.

It has recently been shown[1] that the breaking load of a glass thread depends on the length of the thread solely because the probability of a particularly weak spot is greater in a longer specimen. The statistical treatment of the problem was:

"The breaking load of a specimen L cm long is that of its weakest spot. If this specimen is cut into 10 pieces each $L/10$ cm long, one of these pieces will contain the weakest spot of the original specimen while the remaining 9 pieces will have higher strengths. Therefore the lowest breaking load observed in a batch of 10 specimens of $L/10$ cm should, on the average, be identical with the breaking load of specimens L cm long."

If, for example, the experimental values of the breaking load of the $L/10$-cm specimens are distributed in the "normal" fashion, the probability of the breaking load being between x and $x+dx$ is

$$f(x)dx=\frac{1}{\sigma(2\pi)^{\frac{1}{2}}}\exp[-(x-a)^2/2\sigma^2]dx,$$

σ being the standard deviation and a the mean breaking load. The average value of the smallest breaking load (a_1) in a batch of 10 specimens is therefore

$$a_1=\frac{\frac{1}{\sigma(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{u} x\exp[-(x-a)^2/2\sigma^2]dx}{\frac{1}{\sigma(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{u}\exp[-(x-a)^2/2\sigma^2]dx}. \quad (1)$$

The value of u in Eq. (1) is determined by the condition

$$\frac{1}{\sigma(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{u}\exp[-(x-a)^2/2\sigma^2]dx=0.1.$$

If the long specimen contains n rather than 10 short specimens, the value of u is found from the equation

$$\frac{1}{\sigma(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{u}\exp[-(x-a)^2/2\sigma^2]dx=\frac{1}{n}. \quad (2)$$

Evaluation of the above integrals yields

$$a_1=a-\frac{n\sigma}{(2\pi)^{\frac{1}{2}}}\exp[-(u-a)^2/2\sigma^2]. \quad (3)$$

For $n=10$, $(u-a)/\sigma=-1.282$, and $a_1=a-1.75\sigma$.

Equation (3) shows that the average breaking load a_1 of a long specimen depends not only on that (a) of the short specimen and the ratio (n) of their lengths, but also on the standard deviation (σ) of the breaking load values for the short specimens. It would be interesting to discover a similar relationship also outside the phenomena of rupture.

The statistical theory is, of course, valid only for *some* rupture phenomena. The breaking stress of an adhesive joint is determined by other factors besides the probability of finding a weak spot.[2]

In another, unpublished, study the writer has found that the relation between the persistence (or lifetime) of single foam bubbles and their dimensions cannot be accounted for by the probability theory.

[1] J. J. Bikerman and G. H. Passmore, Glass Ind. **29**, 144 (1948).
[2] J. J. Bikerman, J. Soc. Chem. Ind. **60**, 23 (1941).

A Demonstration Mass Spectrometer

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ANYONE interested in making the explanation of the operation of a mass spectrometer short and clear might be interested in constructing the model described in this article. The apparatus is used as a model of the conventional mass spectrometer but is also a mass spectrometer in its own right.

A junked pin-ball machine was obtained for parts. The sides were cut to 32 in. in length. Since the machine was slightly narrower than could reasonably be tolerated, it was made 34 inches wide. The ejector, feeder, and track were also removed from the pin-ball machine. The remainder of the machine was either junked or dismantled for parts.

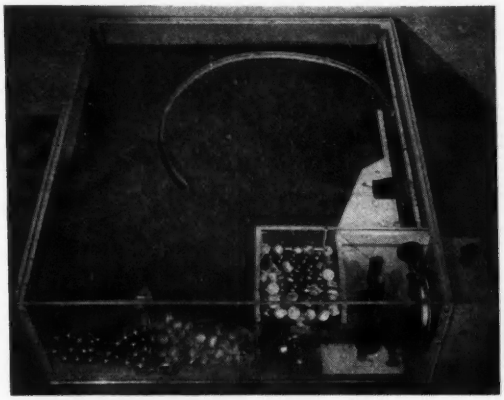

FIG. 1. Photograph of demonstration mass spectrometer.

In Fig. 1, the ball is lifted (feeder) up to the track and fired by a plunger-spring mechanism. Two different kinds of one-in. balls are used, steel and glass. Each ball is given the same amount of energy. The extent to which the helical spring of the ejector is compressed determines the amount of energy imparted to a ball. The ejected ball first travels along a short straight track and is later caused to follow a curved path. A flat steel spring, mounted on its long edge, causes the ball to follow a curved path, and since the spring is free at the far end, the heavier ball follows the path of smaller curvature.

To complete the apparatus, two collectors are placed in the front of the apparatus. The front of the machine is made of Lucite sheet. A Lucite partition is inserted to keep the balls apart once the machine has separated them because of mass differences. To make it convenient for emptying the collector, the back walls slope toward the front, and the bottoms of the collector compartments slope toward the partition. Two holes in the front plate make it possible to remove the balls with ease. A tool clamp is inserted into the holes to keep the balls from rolling out.

We have incorporated an automatic feed and ejector into our apparatus, but this feature is not essential. In brief, the feeder arm is attached to an electromagnetically operated plunger, and the ejector is attached to a motor and speed-reducing mechanism. If the ejector plunger is not automatically controlled, a stop for the ejector plunger should be included as part of the apparatus. This stop is necessary to make certain that each ball gets the same energy.

The wheel (see Fig. 1) which operates the ejector also actuates a microswitch. The contact points of the microswitch are in series with the electromagnet.

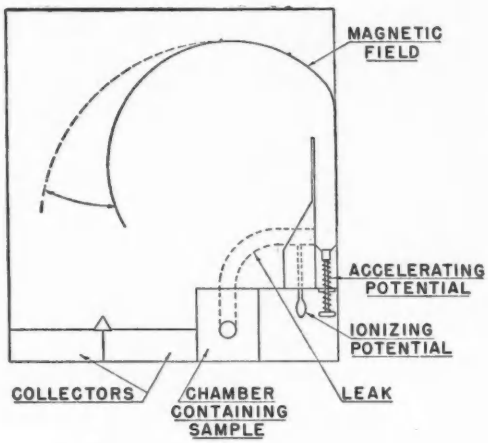

FIG. 2. Schematic diagram of mass spectrometer.

Figure 2 shows a schematic diagram of the apparatus. It is labeled to show the functional correspondence between its parts and the parts of a research instrument.

The Lectures of the Late Professor Benjamin W. Snow

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IN the fall of 1904, when I entered the University of Wisconsin, one generalization requirement for the degree of Bachelor of Arts was a course in some one of the natural sciences. I chose elementary physics and I had excellent reasons for my choice. For the general opinion, expressed with conviction on all sides, was that no student ought to graduate from the College of Letters and Science at the University of Wisconsin without taking Professor Snow's course in elementary physics. Like hundreds and thousands of others, I followed this advice and found that it was sound and rewarding. Only years afterward did I learn that this course was famous throughout the country as an introduction to physics.

Bennie Snow, as he was affectionately known on the campus, was a distinguished physicist, and head of the department; but his favorite course was this introduction to college physics. Like many scholars of his day, Professor Snow had studied at various European universities; he received his doctorate at the University of Berlin in 1892. He was not one of those professors in a graduate school who feel that it is a bit below their dignity to give the elementary course in their field. On the contrary, he lavished time, energy, industry, ingenuity, and enthusiasm on this introductory course.

Physics one, as I believe the course was designated in my time, was not regarded as easy. It brought five hours' credit for each of two semesters. We had four lectures per week by Professor Snow, one searching quiz section each week end under competent quiz masters, and four hours of laboratory work per week. Until the student had acquired an efficient laboratory technique, he rarely obtained the necessary data during the four hours allotted for experimentation; the writing up of the experiments, the sketches of the apparatus, and the statement of conclusions had to be done in additional time.

There were various factors which made for the reputation and the success of the course. Bennie Snow somehow gave the impression that he lived for physics, and that living for physics was a wonderful way to live. His enthusiasm was contagious; he could make even a demonstration of a minor principle of physics seem breathtakingly dramatic, significant, and beautiful. When he explained the acceleration of a sphere rolling down an inclined plane, while he held the sphere all poised at the upper end of the plane, we waited in suspense for the release of the sphere. He had an unusual genius for devising apparatus which illustrated principles of physics in a spectacularly impressive fashion. On entering his large lecture amphitheater with its high ceiling we would be intrigued by the apparatus which was set up for the hour. I recall to this day the geysers, small ones and large ones, which erupted during one of his lectures, each eruption timed with dramatic effectiveness.

Bennie Snow was a natural actor, a remarkable teacher, and a keen psychologist in the field of learning. But although he was a wonderful showman, we never felt that he was merely staging an act. There was far too much sincerity, integrity, thoroughness, and genuine love for science in the man for that. Of course he was entertaining; he could not help but be, yet it was entertainment of a high, sound, educational order that produced pedagogical results which I still feel were decidedly unique. And I recall no falling off of interest on the part of students even after they had heard well over one hundred lectures by him.

Bennie Snow's lectures came at twelve noon on Mondays, Tuesdays, Wednesdays, and Thursdays—a bad hour for lectures and classes as we all know full well. This might have been a particularly bad hour for those students whose first classes had begun at eight in the morning. Yet students very, very rarely went to sleep in these lectures. He had an eagle eye, and if he detected any one asleep, he stopped abruptly and stared with flashing eyes in the direction of the unfortunate sleeper, who came to life almost immediately in the hushed silence. Or if someone whispered to his neighbor, the same thing might happen; but it would happen only once, for the offender felt crushed, even though no word of rebuke was uttered.

Bennie Snow never walked into the lecture hall at the beginning of these lectures. Promptly, when the bell rang at twelve noon he would dash out of his office, run down the corridor, and dart down a short flight of stairs into the amphitheater as if to electrify us all. He would stop his onrush by gripping the heavy slate top of his huge demonstration table, while we all applauded madly with youthful

enthusiasm. Then he would make three bows, one to the center, one to the left, and one to the right, while he beamed affectionately on us with his keen dark eyes and his gracious, dignified smile. When the lecture ended, we again applauded madly, gave a skyrocket cheer for Bennie, and rose while he again, like a great prima donna, made his three formal bows, one to the center, one to the left, and one to the right. Then he would stand between us and his demonstration table, smiling at us as we walked out, and at the same time protecting his beloved apparatus from being jostled. Frequently we speculated on what might happen if we greeted his coming into the lecture hall with silence, but we somehow never had the temerity to do so. We would not have hurt his feelings, even though we had our curiosity about what he might do, or how he might look.

The lectures were carefully organized, the sequence was logical, explanations were concise and couched in careful diction, the progression was not hurried, and there was sufficient repetition to drive points home thoroughly without producing boredom. His definitions were marked by clarity and almost elegant simplicity. As an effective teacher he made frequent use of the blackboard for illustration. He had a well-modulated voice, spoke fairly slowly, and enunciated distinctly. At times the pitch of his voice would rise with dramatic intensity. I do not recall that he lectured from an outline; certainly he read no part of his lectures. We took no notes because we bought a mimeographed set of his lectures which he followed rather closely. Consequently, we were able to devote our undivided attention to him, secure in the knowledge that later on we could refresh our memories from his mimeographed copy.

Vigorous and energetic though he was, Bennie Snow had unusually refined esthetic appreciation. He saw rare beauty in physical nature and in her laws. The regular recurrence of phenomena under controlled similar conditions appealed to him strongly, for, as he asserted, such regularity rules out caprice, and enables man to count on the stability of natural processes in an orderly universe which is subject to uniform law. He could dramatize effectively even so small a thing as an erg; he did this in part through the caressing inflection of his voice, and in part by gestures which never seemed studied; on the contrary, such gestures were spontaneous, for they reflected the genuineness of his interest, his enthusiasm, and his admiration for his subject. For him there was real beauty in the physical universe and in its operation.

One of his favorite special lectures, delivered annually to students and to the general public, was on snowflakes. He was an expert photographer; in this lecture a large number of slides showed the delicate, lacy, hexagonal snowflakes whose subtle beauty and infinite variety never failed to enchant lecturer and audience alike.

It has been my rare good fortune to have many able teachers. But I have known very few who succeeded as well as Bennie Snow in spurring his students on to high effort. If reports came to him from quiz masters that results in the weekly quizzes were not quite satisfactory, he spoke to us so effectively and so warmly in his gentle way that we felt we had hurt him by not doing all that we should. Perhaps we were naive in this particular, yet that was very definitely the effect produced on us by the man's love for science, by his intellectual honesty, and by his kindly attitude toward us.

There were two men on the campus of the University of Wisconsin to whom we tipped our hats in greeting—a mark of real deference on the part of American lads even in those days almost half a century ago. One of them was the distinguished President of the University, Charles Richard Van Hise, an eminent scientist and geologist; the other one was our Bennie Snow. Doubtless more than a few of his students wished that they knew him personally, or that at least he might know their names. But with several hundred in attendance on his lectures each year it was impossible for him to know them as individuals. Yet even so, he left his impress on large numbers of students year in and year out, students who respected his profound interest in his chosen field, and who admired him as a gentleman and as a scholar.

The Lectures of Professor Robert Pohl in Göttingen

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IN an attempt to evaluate the efficiency of different methods of teaching, it is important to consider cases in which one method is used exclusively. It is necessary in this connection to use examples in which the method is used to its fullest advantage. It is with this idea in mind that the present author has attempted to evaluate the famous demonstration lectures of Professor Robert Pohl, which he took about twenty years ago.

In the beginning the background against which Professor Pohl's course is given should be pointed out. It is only a lecture course, without laboratory exercises and without examinations or quizzes. The laboratory is offered as a separate course, in a different department, and is not connected with the lecture course at all. It is indeed not taken by the majority of the students taking Professor Pohl's course. The audience consists of all first-year students in the sciences and in medicine, numbering several hundred. It should be mentioned in this connection that no attendance is taken in German universities, and that students attend or cut classes whenever they see fit. It is therefore indicative of Pohl's success as a lecturer that although he lectured at 7 A.M., he always had a capacity audience. This means that the reputation of his lectures was great, and that the feeling amongst the students was that he was offering information which they could not obtain by the use of textbooks.

The task of Professor Pohl, then, consists in imparting knowledge of physics to students exclusively by the use of lectures and demonstrations without the aid of laboratory exercises, quizzes, or discussion groups. He has solved this problem in such a way that his students obtain from his course a large amount of knowledge and understanding

of physical laws and methods. He accomplishes this by a unique integration of clear, well-considered presentation of concepts and striking experiments.

Since his lectures have been published in English in book form, the general arrangement of the material will not be discussed here. It should be pointed out, however, that the order of presentation of the material, the facts included and excluded, and the manner of discussing them are highly original. Just some examples from the course in mechanics will be given: both in the course on mechanics and in that on electricity Pohl starts out with the theory and methods of making the fundamental measurements; length and time in mechanics, voltage and current in electricity. In the latter case this procedure implies that he omits the time-honored approach through static electricity, and starts out by first introducing the concepts of current and voltage. From the very beginning Pohl emphasizes the necessity of agreeing on a system of reference, and he points out repeatedly in the lectures that in all discussions the system of reference must be specified. In the first lectures, the system of reference is specified to be the floor of the lecture room. The concepts of work, energy, and momentum are introduced under the heading "some useful concepts," since it is pointed out that they permit the quantitative description of certain processes which would otherwise be mathematically difficult to discuss. The gas laws are considered as a special case of hydrodynamics, a gas being treated as a fluid of low density without a surface. Heat is mentioned only incidentally as the sensation of our skin produced by molecular movements.

The most important feature of Professor Pohl's lectures is the very impressive demonstrations. Their outstanding characteristic is their utmost simplicity. Only one experiment stands out in my mind, because it employed a more complex piece of apparatus, a demonstration of the Brownian movement of TiO_2 particles projected through a microscope. Almost all the experiments are done in such a way that the shadow of the apparatus is projected on a screen. In this way even the smaller details are visible to a large audience. There is no lecture table in the classroom, but small tables are set up whenever and wherever needed. Furthermore, it appears important that the number of demonstrations given in any particular lecture is limited to two or three. A large part of the period is devoted to interpreting the experiments in all their possible aspects so as to obtain all the information contained in them.

A particularly impressive feature of Pohl's demonstrations is the extensive use of the human body. In a large number of experiments Pohl uses himself and his assistant as "masses," and it is particularly these experiments which are long remembered by the students. In many instances straight motion is demonstrated by a man on a rolling truck. For example, the law of the conservation of momentum is exemplified by the following simple experiment: A man standing on a truck starts to run on it, imparting an equal momentum of opposite direction to the truck. The truck can be stopped by another man who runs in a direction opposite to the movement of the truck as soon as he steps on it. Finally, a man running all over the truck from one side to the other does not impart any momentum to it.

In experiments on circular motion a rotating stool is substituted for the truck. An example of how this set-up is used to illustrate the concept of angular momentum is the following: Pohl is sitting in the rotatory chair with a gyrostat in his hand, the axis in a vertical direction. Setting the wheel in motion imparts to stool and man a slow rotatory movement in the opposite direction. Braking the wheel by pressing it to the chest stops the motion of the stool. The gyrostat held in such a position that the axis is horizontal does not have this effect. The rotating stool is also effectively used to demonstrate that changes in the position of the human body affect its moment of inertia.

The truck and the rotatory stool are used with particular efficiency in the discussion of accelerated systems of reference. The experiment introducing the concept of Coriolis forces stands out in my mind. On the table of the rotating chair a gun is mounted in such a way that at rest the bullet hits a specified target. When the stool is set in motion, another point of the target is hit. For the observer at rest this means that the bullet flies straight through the lecture room, while the line of sight of the man on the stool rotates. For the observer on the stool, the bullet is deflected by a Coriolis force at right angles to its path. It is then demonstrated that both these statements are mathematically equivalent and lead to the same result. This experiment and others of the same kind, such as the movements of a gravity pendulum on a rotating chair, impress the student with the possibility of describing the same occurrence in alternate concepts. It should be pointed out that experiences of this type appeal very strongly to the students, giving them a sense of intellectual adventure, and a deeper insight into the nature of physical method and physical thinking.

Some words on the mathematical treatment of the experiments should be added. Pohl always insists on expressing the experiments in quantitative terms. This was brought out already by the fact that the whole course is started by discussing the fundamental units of measurement. The mathematical treatment is, however, elementary and involves calculus only occasionally. It is notable that Pohl takes cognizance of the fact that many students' minds are blocked as soon as they see a mathematical formula on the blackboard. Pohl counteracts this block by constantly referring to mathematical procedures by the derogatory term "Kreidephysik" (chalk physics). He succeeds in this way in keeping the attention of the nonmathematical student on the subject even when he uses mathematical methods.

I should like to close by mentioning an experiment in "acoustics" which has been of particular use to me as a biologist. In his discussion of recording apparatus for mechanical vibrations, Pohl uses a simple example for a wrong method: The problem is to record the variation of the blood pressure of a man. This is done by recording the beam of light reflected by a mirror attached to the man's right foot, while his right leg is crossed over his left knee. The mirror is moved by the pulsation of the artery in the knee, the leg serving as a lever. But, except for the period of pulsation the curve obtained is completely wrong, due to the resonating properties of the lever and the phase

difference between the resonator and the exciter. This warning is most useful for a biologist to understand.

In conclusion it should be stated that Professor Pohl's lectures show that it is possible to teach beginners in physics at a rather advanced level. It is furthermore notable that this can be achieved successfully by means of lecture and demonstration only. This success appears to be due to the fact that Pohl has broken in many respects with traditional and historical ways of presenting ideas, and that he has devised demonstration experiments which impress the student and are retained by him, particularly because of the simplicity of the apparatus used. It appears from these considerations that the demonstration lecture is one of the most powerful teaching methods. Its success seems to be particularly dependent on the close integration of experiment and conceptualization.

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Letters to the Editor

The Weight of a Falling Chain

A CLASSICAL and intriguing problem in analytical mechanics is the case of a falling chain. A uniform chain of length L and total mass M is held vertically with its lower end just touching a balance pan. The fixed upper end is released and the chain "accumulates" on the scale pan. What is the maximum reading of the balance?

Although I have published elsewhere[1] the purely analytical problem it has just occurred to me to investigate the experimental behavior. This note is intended to report a most interesting observation.

It is easily shown analytically that the maximum reading of the balance is $3M$, that is, that the force on the platform is, at any instant, three times the weight of the chain already fallen. Interestingly, the analysis shows that the force delivered to the pan is composed of two things, the weight of the chain already there (say w) and a force due to impact which is always $2w$. The scale reading, then, grows from zero to $3M$ and instantly decays to M.

Now in the investigation of this experimentally (using an old-fashioned toilet pull-chain) I used first a spring scale with dial face and second a platform beam balance. With the spring scale the maximum reading was indeed 3 times the static weight of the chain. With the platform beam balance the maximum reading was 2 times the dead weight of the chain. How shall we resolve this dilemma?

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[1] J. S. Miller, Math. Mag. 21, 159 (1948).

The Energy Density in a Gravitational Field

WHEN two oppositely charged bodies are attracted together we perhaps recognize that this energy is derived at the expense of energy in the electric field. We are less ready to recognize energy as located in the gravitational field. We are likely to say that the energy of a ball remains constant as it falls toward the ground although we must recognize that work is done upon it (by the gravitational field), that its mass increases, and that mass is a manifestation of energy. The principle of continuity gives a meaning to the localization of energy. If the energy and mass of the falling body increases while the energy of the entire system remains constant, energy must reside in the field. Miller has recently given the expression $\mathbf{g}^2/8\pi G$ for the energy density of the gravitational field[1] and asks whether his analogy with the electromagnetic field is justified.

In ordinary gravitational theory we are concerned with the interaction between bodies with velocities small in comparison with the velocity of light and so the gravitational field is to be compared with the electrostatic field. The gravitational field near two mass particles resembles the electrostatic field of two like charges but the force is like that between two opposite charges. In consequence of this the energy of the field is negative. The expression for the energy density is derived just as in electrostatics; the difference in sign occurs because $\mathrm{div}\mathbf{g}=-G4\pi\rho_m$ while $\mathrm{div}\mathbf{E}=k4\pi\rho_e$. If φ represents the gravitational potential, the potential energy variation δU_p due to a density variation is

$$\delta U_p=\int\varphi\delta\rho_m d\tau=-\frac{1}{4\pi G}\int\phi\,\mathrm{div}\delta\mathbf{g}d\tau.$$

After integration by parts the expression becomes

$$\delta U_p=\frac{1}{4\pi G}\int\mathrm{grad}\varphi\cdot\delta\mathbf{g}d\tau=-\frac{1}{4\pi G}\int\mathbf{g}\cdot\delta\mathbf{g}d\tau$$

and

$$U_p=C-\frac{1}{8\pi G}\int\mathbf{g}^2d\tau.$$

If we wish to ascribe an absolute value to the energy, C must be determined. In a typical case we shall be interested in the energy of two rigid bodies, for example the earth and a ball. The potential energy is the energy of interaction as their separate fields $\mathbf{g}_1$, and $\mathbf{g}_2$ combine to give $\mathbf{g}$. For infinite separation $U_p=0$ and since the fields do not overlap

$$U_p=C-\int(1)-\int(2)=0,$$

where the symbol $\int(n)$ represents the negative self-energy integral of the nth particle alone $\int(\mathbf{g}_n^2d\tau)/8\pi G$.

Thus, for the mutual potential energy of two particles

$$C=\int(1)+\int(2)$$

and

$$U_p=\int(1)+\int(2)-\int\frac{\mathbf{g}^2d\tau}{8\pi G},$$

which is always negative. If the particles are spheres of uniform density the constants $f(n)$ have the explicit values $\frac{3}{5}Gm_n^2/r_n$. The electrostatic result is similar except for sign. The potential energy of two electrons is

$$U_p=\int\frac{\mathbf{E}^2d\tau}{8\pi k}-2\int(e),$$

where $f(e)$ represents the energy integral for the isolated electron $f(\mathbf{E}_0^2d\tau)/8\pi k$. If the charge is assumed to be distributed uniformly throughout a sphere, $f(e)=\frac{3}{5}kq^2/r$; if it is distributed over the surface, $f(e)=\frac{1}{2}kq^2/r$.

Consider two particles with rest masses m_1, m_2 and actual masses m_1, m_2, where $m=m/(1-v^2/c^2)^{\frac{1}{2}}$. The first may be the earth and the second a ball. The energy of the system consists of the self-energy of the particles $m_1c^2+m_2c^2$ and their mutual energy U_p which depends upon their separation

$$U=m_1c^2+m_2c^2+U_p\backsimeq m_1c^2+m_2c^2+\tfrac{1}{2}m_1v^2+\tfrac{1}{2}m_2v^2+U_p,$$

where the last term is negative. As the particles fall together, their individual energies m_1c^2, m_2c^2 increase, while U_p, the energy of the field, decreases. (The integral $f\mathbf{g}^2d\tau$ increases.) As the ball falls to the ground, its energy and mass increase as it sweeps up energy from the gravitational field. The energy of the system U and the mass of the system U/c^2 remain constant.

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[1] J. S. Miller, Am. J. Phys. **18**, 237 (1950).

Does Pressure Have Direction?

IN recent discussions in these columns[1] as to whether pressure is a scalar or a vector, difficulty appears to have arisen from a lack of generality in treatment. In elementary discussions, pressure is defined as the force per unit area at a point; if, as is usually the case, the pressure varies from point to point, one would have to make such a definition as

$$p=\text{limit}_{A\to 0}(F/A). \tag{1}$$

In advanced work, F is a vector, and so is A, the direction of A being taken as that of the normal to A, outward or inward as desired, as long as one adheres to a single definite convention. But then $p=\text{limit}_{A\to 0}(\mathbf{F}/\mathbf{A})$ is meaningless, as division is not defined for vectors. One must then write: In the limit as $\mathbf{A}\to 0$, p is defined by the equation

$$\mathbf{F}=p\mathbf{A}. \tag{2}$$

In an ideal (nonviscous) fluid, or in any fluid at rest, $\mathbf{F}$ and $\mathbf{A}$ have the same direction, so that p may be treated as a scalar. In the general case, the force exerted by a fluid upon an element of area is not normal to the surface, so that we should write something like

$$\mathbf{F}=\mathbf{\Pi}\cdot\mathbf{A}, \tag{3}$$

where $\mathbf{\Pi}$ is a linear vector operator or tensor. In the special case of a fluid at rest, or of an ideal fluid,

$$\mathbf{\Pi}=p\mathbf{I}, \tag{4}$$

where $\mathbf{I}=(\mathbf{ii}+\mathbf{jj}+\mathbf{kk})$ in rectangular coordinates, and is the *idemfactor*. Then

$$\mathbf{F}=\mathbf{\Pi}\cdot\mathbf{A}=p\mathbf{I}\cdot\mathbf{A}=p\mathbf{A}, \tag{5}$$

as the dot-product of any vector by the idemfactor is the original vector.

Teachers of physics are undoubtedly responsible for confusion in the minds of students, for many elementary texts contain such statements as "the pressure in a fluid at rest is the same in all directions at any given point, and is perpendicular to any surface upon which it acts." I am sure I have said something like this many times. A correct statement would be: "In the case of a fluid at rest, the force exerted by the fluid upon any surface is perpendicular to that surface; at any given point, the force exerted upon an element of surface is independent of the orientation of that surface." Whether the average freshman could understand the difference between these statements is another question.

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[1] R. D. Summers, Am. J. Phys. **14**, 311 (1946); **17**, 319 (1949). Vola P. Barton, Am. J. Phys. **17**, 318 (1949). A. R. Stickley, Am. J. Phys. **18**, 322 (1950).

Motion of a Particle through a Resisting Medium of Variable Density

THE motion of a particle through a resisting medium of constant density is treated in most of the well-known textbooks of mechanics. However, many interesting problems involve the motion of a particle through a resisting medium whose density is a function of position along the path.

Consider the fall of a particle dropped from rest from a great height through the atmosphere, whose density diminishes exponentially with the altitude; let the origin be taken at the point from which the particle is dropped, and let the distance traveled be denoted by y; finally, let the resistance be proportional to the product of instantaneous speed and air density. Then, the equation of motion of the falling particle may be written

$$\ddot{y}=g-ke^{\beta y}\dot{y}.$$

Or, if the resistance is proportional to the product of the square of the instantaneous speed and air density, the equation of motion is

$$\ddot{y}=g-ke^{\beta y}(\dot{y})^2.$$

One can easily obtain for these equations solutions containing unfamiliar, complicated, power series expansions.

Query: Is it possible to obtain solutions in closed form?

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ANNOUNCEMENTS AND NEWS

Book Reviews

Methods in Climatology. V. CONRAD AND L. W. POLLAK. Pp. 459+xxvii, Figs. 115, Tables 101. Second edition. Harvard University Press, Cambridge, 1944. Price $7.50.

This is an invaluable textbook for the serious student of climatology who must start with the observable materials, such as the climatic elements, and deduce a quantitative description of a climate.

Part I is composed essentially of the material given in the entire first edition, together with the addition of a number of new items. It is concerned with methods that are useful primarily in representing climatological phenomena and solving climatological problems. Many of the methods that are described and illustrated by means of examples from climatology and meteorology are equally applicable to problems in geology, geography, geophysics, certain branches of agriculture, and to many fields of physics.

In Chapter 2, the treatment of statistical analysis of climatic elements is excellent and the methods given could be followed in the analysis of the data obtained in many of the research problems in physics.

Chapter 3 is devoted to such aids to computation as graphs, models, monographs, computation charts and alignment charts, while Chapter 4 is devoted to curve fitting and smoothing of numerical series. Here again the principles given and illustrated have a wide application to all fields of quantitative research. The careful study of these two chapters would be of great value to any student entering any field of physical research.

In Chapter 5 the use of Fourier's series in harmonic analysis is fully explained and illustrated. Some special procedures of analysis by the authors are presented. The application of harmonic analysis to the diurnal variations of the climatic elements is discussed and illustrated.

The remaining chapters of Part I are devoted to the consideration of the characteristics of some selected elements and to the spatial comparison of climatic elements.

Part II, which is entirely new, begins in Chapter 11 with a discussion of some mechanical and electrical devices designed to handle large numbers of observations without regard to their sources and consequently can be used to handle climatological, geophysical, biological, or physical data. Included in this chapter is the description of punch card machines and their special advantages for handling climatological and meteorological data are demonstrated.

Most of the remainder of Part II is devoted to the discussion of meteorological and climatological cycles and their periodicities. Theory and methods are given for revealing hidden periodicities and for testing the realities of the results obtained. The last chapter presents Fuhrich's autocorrelation method, which seems to be superior to earlier attempts.

This textbook is highly useful in a wide variety of fields; while it is invaluable to students and scholars of climatology, meteorology, and geophysics, it is also quite useful to students of physics, biology, zoology, agriculture, geography or of any science or activity where the analysis of data is important.

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The Autobiography of Robert A. Millikan. Pp. 311+xiv. Prentice-Hall, Inc., New York, 1950. Price $4.50.

The *Autobiography of Robert A. Millikan* is one of the most outstanding works of its kind done by an American man of science. The treatment is lucid and brings out in clear relief not only the activities of the man himself but of those, and there are many, with whom he has associated and collaborated in the fields of teaching, research, and administration.

The autobiography is that of a dynamic personality associated with patience, persistence, and enthusiasm. The treatment is free from egotism and refreshingly frank and forthright. One may regard Professor Millikan's activities as divided into three periods not, however, without overlapping. First, his period of training, teaching, and research extending to the time of his appointment to a professorship at the University of Chicago. Second, the period beginning roughly with World War I and ending when he left the University of Chicago to take over the administration of the California Institute of Technology. Third, the period involving both academic and extra-academic activities in Pasadena.

During the first period his efforts appear to have been devoted largely to teaching, textbook writing, and research. One must admire the wide range of his research activities and the persistence with which they were carried out. The work for which he is best known, and for which he received the Nobel Prize in 1923, is his oil drop experiment which led to an accurate measurement of the charge on the electron. His work on the photoelectric effect dealing with the verification of the Einstein equation, $hf = Ve = \frac{1}{2}mv^2 + \omega$, was no ordinary undertaking. The account of his researches during this period is intriguing. The many friends he made during this period in both America and Europe in effect provided the momentum which carried him over into the second period. His broad training in fundamental physics along with his dynamic personality paid dividends, not only in the research required for the war effort, but also in the administrative duties into which he was ushered. His fostering of an extended American Academy of Science, of the creation of the National Research Council and, above all, his opposition to a single super-research Institute, along with the advocacy of National Research Council Fellowships are to his everlasting credit. As one follows his activities in this period, one is not only impressed with their variety but also with the physical stamina of the man, as well as his vigor.

The decision to leave the University of Chicago to enter upon his duties at the California Institute of Technology was not an easy one to make. It is interesting to insert the quotation of Professor Michelson found on p. 217. "If I were a few years younger, I would join you in an ultimatum that it is a substantial increase now in the physics budget or else." These are startling words from a great scientist in a great university. How many times have similar remarks been made by others in other institutions! How often have indifference and lack of understanding on the part of the administrators of our academic institutions been responsible for the loss of fine scholars! Professor Millikan, on p. 219, modestly indicates that the best thing that ever happened to physics at Chicago followed his departure, meaning of course, that the University decided to rectify the error by bringing to it younger and vigorous scholars.

His first task at the California Institute of Technology was to develop an outstanding department of physics. His success is evidenced by the number of National Research Council Fellowships, the lack of which appeared to be the provoking element in his decision to leave Chicago. The development of a strong department of physics followed from his views with regard to the place of physics among the sciences. On p. 43 it is remarked that "··· has convinced me that every man and woman should and can easily know about physics for it is unquestionably the most basic of all sciences and the one upon which they all depend." To defend such a thesis in the face of other sciences requires courage. At the Institute a strong department of physics meant also strong departments of technology.

His organization of the administration of the Institute was unique in the United States. It followed somewhat the pattern of many European universities. The administration was carried on largely by the Executive Council consisting of four prominent professors and four prominent trustees, the chairman of which was Professor Millikan. There was no galaxy of administrative officers to ensnare the professors in red tape.

The last three chapters are devoted to an exposition of his political, social, economic, and religious views. The views have not been confined wholly to these chapters for they emerge throughout the volume. Prior to the turn of the century a remarkable and turbulent era began in science, particularly in physics. The effects of discoveries made and theories propounded by such men as J. J. Thomson, Rutherford, Planck, Roentgen, Einstein, Laue, and others following the year 1890 have had a profound effect upon the social and economic structure of our civilization, effects which may make or destroy it. It was at the beginning of this period that Professor Millikan began his career as a physicist. He has played his part in shaping our destiny. There are those who may differ from his views, but his maturity and wide experience as a scientist and man of affairs are cogent reasons for a careful perusal of those chapters by persons in all walks of life. Had the views he expressed in the chapter on "The Roads to Trace" been followed after World War I, the world would not be in the mad frenzy of today which threatens the world with disaster.

For the reader who is a contemporary or near contemporary, the volume will be found refreshing in that it brings into relief not only the activities of the author, but also of the many with whom he was associated and for whom he always says a kindly word.

For the younger generation of scientists the book should serve as a handbook of good advice. It indicates clearly what patience, hard work and clear thinking can accomplish in a system of free enterprise.

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Reflections of a Physicist. P. W. BRIDGMAN. Pp. 391. Philosophical Library, New York, 1950. Price, $5.00.

This book presents most of the nontechnical articles and addresses prepared by Dr. Bridgman during the last quarter of a century. The operational approach permeates nearly all of them. In this approach, he assumes that "the meaning of any concept is to be sought in the operations, whether physical or mental, which are performed in making application of that concept."

The first half of the book deals mostly with physical applications of operationalism. For example, in "The New Vision of Science," Bridgman offers "the conviction that the world is understandable is civilization's most important gift to mankind." Nevertheless he feels the recent scientific revolution has convinced us that in the atomic world nature is *not* understandable and *not* subject to law. Cause and effect do *not* operate.

The second half of the volume is devoted mostly to science and society. In "The Struggle for Intellectual Honesty," Dr. Bridgman points out that increased freedom from toil, and universal education have permitted more people to indulge in intellectual pursuits. Moreover, "in scientific activity, the necessity for continual checking against the inexorable facts of experience is so insistent··· that a high degree of intellectual honesty is the price of even a modest degree of success."

Some of the immediate results of the new widespread zeal for intellectual integrity are bad. The scientist finds himself incapable of forcing upon his children "the positive sanctions for conduct that are so necessary in the young" ···"the incidence···of a passion for···intellectual honesty must, at first, be entirely negative and destructive."

In "The Strategy of the Social Sciences," he claims that the social sciences are more akin to engineering than to pure science. Progress must be slow because of the difficulty of performing fundamental, controlled experiments. Then, too, the language of social science is shot through with conflicting rational and emotional meanings. "It may take thousands of years to get anywhere. It took physics two thousand years to reach its present position."

Bridgman, like many others, is most unhappy at the status of the intellectual in this "Century of the Common Man." He regrets the passing of the day when most research was free from administrative duties and from government controls. He resents the iron curtain so fiercely that he refuses to admit to his laboratory any visitor from a totalitarian nation. He detests pressure groups that get more than their share of the "gravy." To him, a philosophy

of democracy that men are of equal value and therefore must be treated alike is blind to the "undemocratic distribution of talents by nature." "It is important for everyone that unusual ability, so long as it is accompanied by social responsiblity, be cherished, encouraged, stimulated, and given any special treatment necessary for it to produce to capacity."

This book, written by one of America's few outstanding physicist-philosophers will repay study by every physicist-teacher.

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First Principles of Atomic Physics. R. E. HUMPHREYS AND R. BERINGER. Pp. 390+ix, Figs. 186, Tab. 6, 15×23 cm. Harper and Brothers, New York, 1950. Price $4.50.

This book is planned as a terminal course for students of the humanities and social sciences. Approximately one-third of the book (126 pages) is elementary mechanics and heat. There follow 70 pages of electricity and magnetism, while the remaining 182 pages including the appendix are devoted to atomic and nuclear phenomena. At the conclusion of each of the 29 chapters there is a selected list of problems, also references to suggested reading; illustrations are ample.

The expressed aim of the authors is to keep mathematics at a minimum, "yet one must not present a course without mathematics since this would seriously distort the physicist's methods and sterilize the logic of physics." In general, the authors have used discriminating judgment in their choice of mathematical material, deriving the formulas in the simplest acceptable way. Among the exceptions may be noted the algebraically clumsy derivation of the standard formula $v^2=v_0^2+2as$ (p. 30). The derivation of the pressure of a gas from the kinetic theory of gases (pp. 101–104) is doubly unfortunate. There we see an attempt to give a more precise derivation than is ordinarily presented in elementary or even general physics textbooks by taking into account the range of molecular speeds instead of assuming a definite root-mean-square value. The derivation thus becomes much too involved for a text with so humble an aim. Whatever rigor is gained by this treatment is more than lost when the molecular forces in the three axes are added arithmetically as scalar quantities,—a fault which could have been easily avoided. In some other cases relatively simple derivations are omitted. There is no derivation for the simple harmonic motion formula or for the simple pendulum, though there is an effort to make the formulas seem reasonable.

On p. 108 there is a serious error of fact wherein the authors take the Boltzmann formula for the kinetic energy of translation of gas molecules as their total energy. Thus they obtain the total energy of a gas at 294 degrees K as 3.66×10^3 joules per mole. "This is the amount of energy that would be needed to heat up a mole of gas (observe that it does not matter what kind of gas) from absolute zero to room temperature," a statement which says in effect that the specific heat of every gas is three calories per mole!

The authors have developed a pleasing style which made the reading of the book by the reviewer an enjoyable experience. In many cases the phraseology is distinctively their own. However, their unique definitions and statements sometimes detract from rather than add to the clarity of conventional statements, e.g., defining the amplitude of a wave as "its maximum disturbance" (p. 118). Their avoidance of the word *torque* can only lead the student into confused thinking: "A coil of many turns is employed to increase the rotational force for a given current" (p. 173). *Torque* is nowhere introduced into the text.

In the second part of the book the electrostatic system of units is quite carefully developed. The coulomb, volt, and ampere, named as practical units, are undefined but are evaluated in terms of electrostatic units. Magnetic field is actually defined in terms of electrostatic units: "The unit magnetic field strength is the field in which a current of one esu flowing in a conductor one centimeter long (and at right angles to the field) experiences a force of one dyne" (p. 169). Then "the oersted is $1/3\times10^{10}$ esu of magnetic field." There is no reference to electromagnetic units as such. All problems involving magnetic fields are in esu. Since the primary interest in magnetic fields centers in their effect on the motion of charged particles this approach will probably meet with approval.

The third and main part of the text is written on a high plateau of excellence. Any student about to enter a standard course in atomic and nuclear physics could read this part with profit. He will find the treatment stimulating though necessarily condensed. The reader is carried through the work of J. J. Thomson, through Millikan's determination of the charge of an electron, the separation of isotopes, the Michelson-Morley experiment, the shift from Galilean to relativistic transformations, mass-energy equivalence, the birth and growth of the quantum theory, the Bohr atom, the Compton effect, particle waves, the uncertainty principle, Rutherford's theory of scattering, induced transmutation of the elements, the discovery of the positron, the neutron, and the meson and the evidence for the existence of the neutrino, induced radioactivity, particle accelerators, nuclear fission, nuclear fusion, and many other topics, each accompanied by a reasonable measure of mathematical development.

The order of presentation is not that of conventional textbooks nor is it the historical order; however it seems to suit the purposes of the authors very well. In any case, the development cannot be made simple as long as the authors are not content with purely superficial treatment. In the opinion of the reviewer the material presented represents the minimum knowledge of nuclear physics which the broadly educated world-citizen of today should possess.

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The Nature of Physical Reality. A Philosphy of Modern Physics. HENRY MARGENAU. Pp. 479+xiii. Figs. 13, 6¼×9¼ in. McGraw-Hill Book Company, Inc., New York, 1950. Price $6.50.

For ages the problem of reality and the existence of a real world has proved a happy hunting ground for pro-

fessional philosophers, and physicists have not hesitated to join the hunt on occasion. In more recent times with the rise in the influence of logical positivism in its various forms on the methodology of physics, there has developed a tendency to consider reality as a pseudo-problem of no consequence to the working physicist. In the significant volume under review Professor Henry Margenau takes definite issue with this trend. He definitely believes that "a clear, though perhaps only a partial understanding of what is real" is possible, and it is his aim to provide it through the agency of a careful elucidation of the method of physics. Those who are acquainted with Professor Margenau's numerous contributions to physical methodology will be greatly stimulated by this excellent summary of his views, even when they are unable to accept in every respect the results of his reasoning. The book is written in the clear, lively, and persuasive style for which the author is well known. Since in the development of its principal thesis it involves a thorough summary of the theoretical structure of both classical and modern physics with a minimum of elaborate mathematical analysis, it should prove of great value to all students of physics who wish to get a broader perspective of their science and for once see the woods as well as the trees.

The author begins by candidly admitting that one can pursue scientific investigations without holding any particular view about reality or indeed any view at all. However, he feels we cannot hope to understand the methodology of science without excursions into metaphysics and it is here that we must face courageously the problem of reality. Indeed it is precisely only in a thorough study of physical methodology that we can expect to understand what physical reality means. Therefore the author embarks on such a study, beginning with a cursory review of sense data and proceeding to the necessity for the introduction of rational constructs, i.e., concepts like external object, material particle, mass, energy, field vectors, and the like —the physicist's stock in trade.

The importance of rational constructs is documented by the listing of numerous metaphysical requirements they must satisfy, including logical fertility, simplicity, the existence of multiple connections between them and sense data, and so forth. It is only through their validation by experiment and measurement that they gain their real significance for physics, and hence the author includes a chapter on precision of measurement. Of all physical constructs those of space and time have long appeared as in many respects the most fundamental, and this suggests a careful comparison of the modern views of Poincaré and Einstein with the classical notions of Kant and Newton.

The positivistic thesis that a science like physics *describes* but does not *explain* is not congenial to Professor Margenau. He believes that both elements enter: when constructs lie near the plane of perception, i.e., are very closely connected with raw sense data, we are justified in considering the resulting theory as descriptive, whereas when constructs are more removed from everyday experience we may plausibly talk of explanation. From this viewpoint the relativity theory of gravitation and the theory of quantum mechanics are more truly explanations of the associated phenomena than Galilean mechanics is of the ordinary motion of terrestial bodies. Many will consider this a somewhat arbitrary distinction, though it does emphasize a direction in which modern physics is tending.

The author's brief survey of classical mechanics and thermodynamics is more or less conventional, but does provide him an opportunity to pay more attention to the problem of the definition of physical constructs than is usual in books of this kind. Most teachers of physics have insisted that there shall be one and only one definition of each construct, but Margenau points out that research physicists do not actually proceed in this way. Rather there exist two large and essentially different classes of definitions which are necessary for the assignment of physical reality to any particular construct. The *epistemic* or operational definition indicates the connection between the construct and sense data, whereas the *constitutive* or theoretical definition relates the construct to other constructs in the same theory. Thus if the construct is a quantity like mass, the description of the measurement of mass by means of a balance is epistemic, whereas Newton's second law (or equivalent) is the constitutive definition. The point is argued with much cogency; the reviewer, however, is inclined to the opinion that logical clarity would be furthered by the requirement that a single definition is sufficient, but that it involve both epistemic and constitutive elements. This doubtless is a question of taste as are in the last analysis most problems of physical methodology.

In his fifteenth chapter the author finally comes to grips with the key question: what really constitutes physical reality? The answer there suggested will not surprise the reader who has carefully digested the earlier parts of the book. Physical reality means the existence of *valid* constructs, i.e., constructs which have met all the metaphysical requirements and at the same time have survived experimental verification. But this view admittedly raises some difficulties. If a construct is created today and validated, was it real yesterday? May a real construct cease to be real? Presumably according to the author's thesis the answer to both questions is in the affirmative. The reviewer has long felt that a concept of this kind, though an interesting philosophical article of belief, is of questionable pragmatic value in the actual development of physical science. But here again the refrain must be "*de gustibus*···."

The reader could hardly expect to find anywhere a clearer and more elegant exposition of the fundamentals of quantum mechanics that Margenau provides in his final six chapters. Of particular importance is the insistence that the indeterminacy principle is a direct consequence of the quantum-mechanical definition of state and not simply a matter of the interaction between a measuring device and a system. The philosophical importance of the Pauli exclusion principle is given more than usual stress and its bearing on the mysterious "exchange" forces of quantum mechanics explained in convincing fashion.

Every student of physics will gain a deeper insight into the meaning of his science by a thorough reading of this stimulating book.

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